CS 130, Final

Solutions

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Σ

Read the entire exam before beginning. **Manage your time carefully.** This exam has 50 points; you need 40 to get full credit. Additional points are extra credit. 50 points → 2.2 min/point. 40 points → 2.8 min/point.

Problem 1 (2 points)

Concrete sidewalks are generally observed to be very brightly lit by the sun at around noon but only dimly lit when the sun is near the horizon. Why? You may assume that the sun is unobstructed (not shadowed by trees, mountains, or other objects that may extend above the horizon).

This occurs for the same reason as in the Lambertian lighting model. When the sun illuminates the sidewalk at a grazing angle, an incoming cylinder of light rays will be spread over the sidewalk over a much larger area. Since less light strikes a given area of the sidewalk, it appears dimmer.

Problem 2 (2 points)

Which of these dominates the cost of ray tracing when acceleration structures are not in use and why? (a) Computing rays to cast, (b) calculating surface normals, (c) shading computations, (d) intersections, or (e) texture mapping.

If there are p pixels, n objects, and l lights, ignoring reflections or transparency, the operations above will execute approximately (a) $p + pl$, (b) p, (c) p, (d) $pn + pnl$, (e) p times. Of these, intersections are the most expensive.

Problem 3 (2 points)

Construct a one-to-one function $f(x)$ mapping from $[a,b]$ to $[c,d]$.

$$\begin{aligned}
f(x) &= rx + s \\
c &= ra + s \\
d &= rb + s \\
(d-c) &= r(b-a) \\
r &= \frac{d-c}{b-a} \\
s &= c - ra \\
s &= \frac{cb-ad}{b-a} \\
f(x) &= \frac{d-c}{b-a}x + \frac{cb-ad}{b-a}
\end{aligned}$$

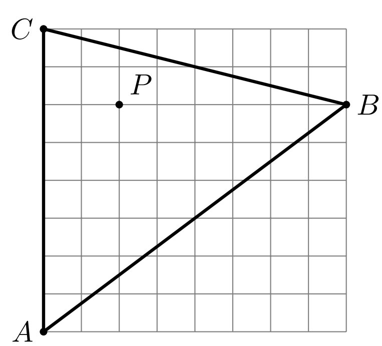
Problem 4 (2 points)

What geometrical transformation does this 2D homogeneous transformation matrix correspond to? $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

It scales by 1/3 in the x direction and 2/3 in the y direction. Note that it scales the w by 3, so that when the perspective divide is performed, this 3 is divided from the x and the y.

Problem 5 (3 points)

Find the barycentric weights for the point P in the triangle below.



To do this, we need to compute the areas of the triangles, which we can then use to compute barycentric weights. Note that all of the triangles we need have an edge that is horizontal or vertical, so $A = \frac{1}{2}bh$ is easy to compute.

$$\text{area}(ABC) = 32 \qquad \text{area}(PBC) = 6 \qquad \text{area}(APC) = 8 \qquad \text{area}(ABP) = 18$$

$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{6}{32} \qquad \beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{8}{32} \qquad \gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{18}{32}$$

Problem 6 (4 points)

The equation $2z = x^2 + y^2$ represents a paraboloid. Compute *all* of the intersection *locations* between this paraboloid and the ray with endpoint $\mathbf{e} = \langle 0, -1, 3\rangle$ and direction $\mathbf{u} = \langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\rangle$.

$$\begin{aligned}
x &= \frac{t}{3} \\
y &= \frac{-3 - 2t}{3} \\
z &= \frac{9 + 2t}{3} \\
2z &= x^2 + y^2 \\
2\left(\frac{9 + 2t}{3}\right) &= \left(\frac{t}{3}\right)^2 + \left(\frac{-3 - 2t}{3}\right)^2 \\
6(9 + 2t) &= t^2 + (3 + 2t)^2 \\
0 &= 5t^2 - 45 \\
t^2 &= 9 \\
t &= \pm 3
\end{aligned}$$

The negative solution does not lie on the ray, so $t = 3$. Plugging this in we get the intersection location $(1, -3, 5)$.

Problem 7 (4 points)

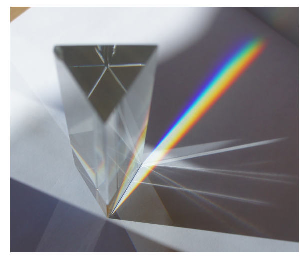
The equation $2z = x^2 + y^2$ represents a paraboloid. Given a point (x, y, z) that is on the paraboloid, compute the normal direction at that location.

This can be done easily as a parametric equation: $(u, v, (x^2 + y^2)/2)$. It can also be done as an implicit equation: $f(x, y, z) = x^2 + y^2 - 2z = 0$. The latter approach is simpler, so we will do it that way.

$$\nabla f = \begin{pmatrix} 2x \\ 2y \\ -2 \end{pmatrix}$$

$$\mathbf{n} = \frac{\nabla f}{\|\nabla f\|} = \frac{1}{\sqrt{x^2 + y^2 + 1}} \begin{pmatrix} x \\ y \\ -1 \end{pmatrix}$$

Problem 8 (2 points)

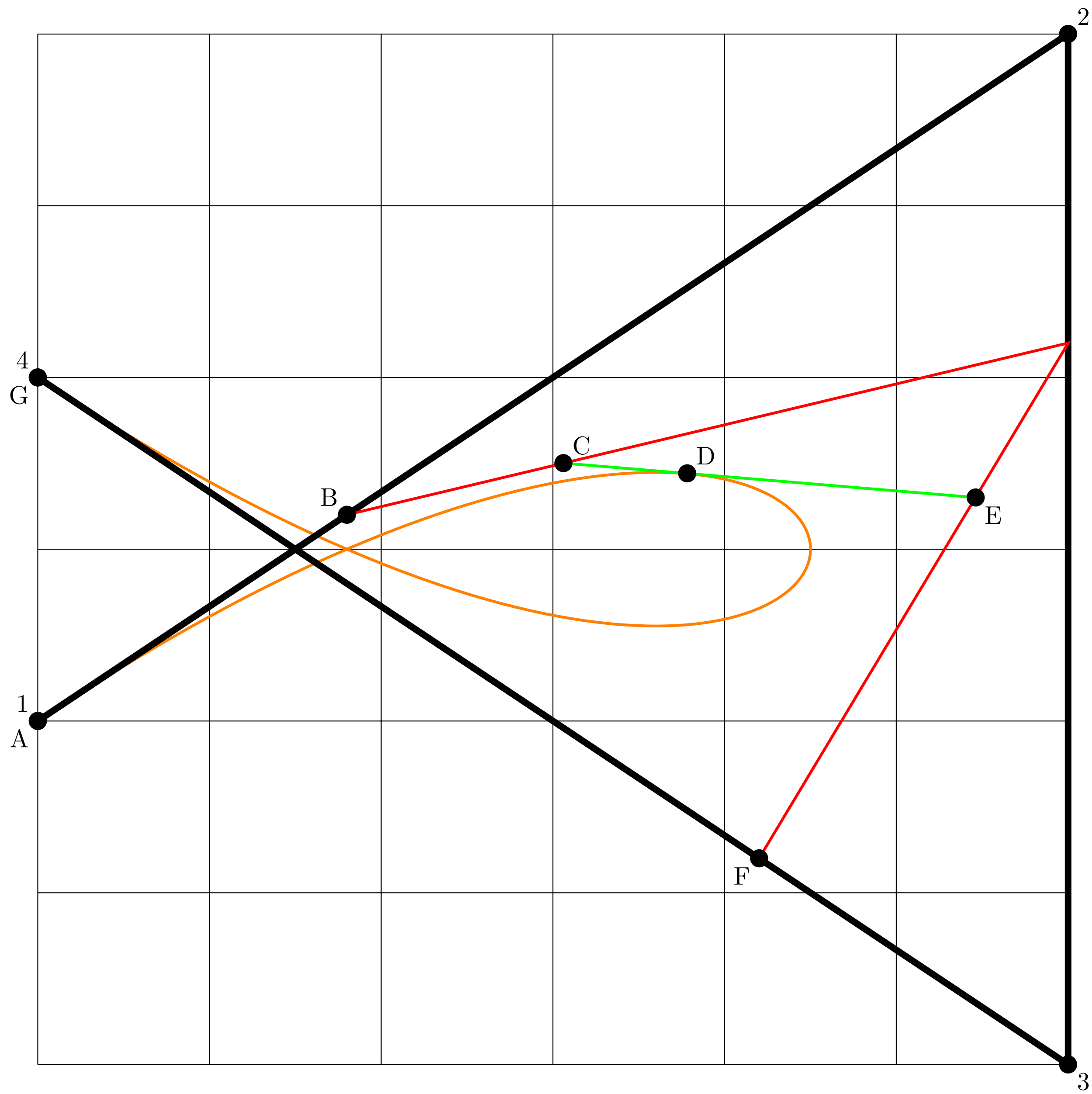


A prism is a transparent object that has different indexes of refraction for different wavelengths of light. As a result, for example, red and yellow light bend by different angles as they pass through the prism. In this way, sunlight is divided into a rainbow upon passing through a prism. (See the photograph to the right.) What would be observed if light from a computer screen displaying a solid white background were passed through the prism instead of white light from the sun?

Instead of a rainbow, you might see colored bands for red, green, and blue (in the same places that they occur in the rainbow) with no light between those bands. If the red, green, and blue are not pure, a few bands are observed. A rainbow will not be observed. (For example, sunlight contains light that is actually yellow. Yellow in a computer monitor is red plus green.)

Problem 9 (4 points)

Geometrically subdivide the Bézier curve given by the control points below and at $t = 0.3$ along the curve. Label the new control points A, B, C, ..., G. (The two resulting Bézier curves should share their common point, so only 7 control points are required.)



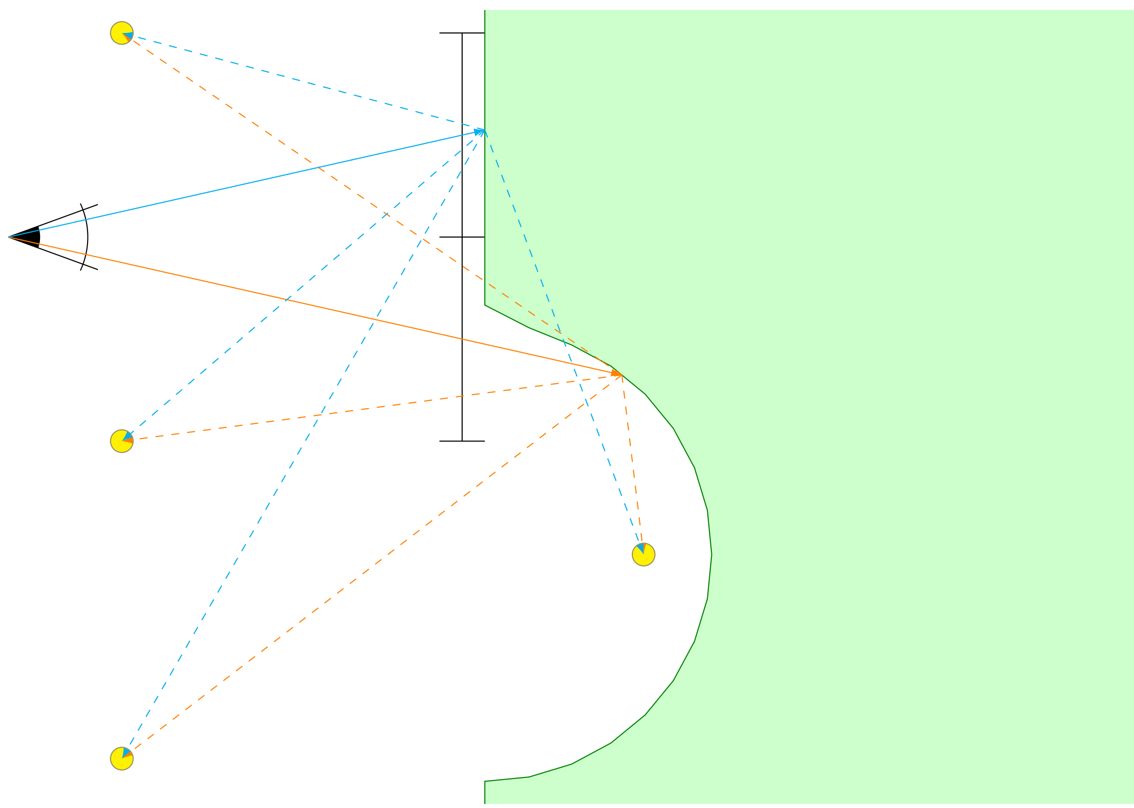
Problem 10 (2 points)

What are homogeneous coordinates and why does the graphics pipeline use them?

These are coordinates with an extra component (x, y, z, w) that are related to regular coordinates by $(x/w, y/w, z/w)$. The extra coordinate makes it possible to represent translations and projections as a matrix.

In the raytracing problems below, green objects are wood, red objects are reflective, and blue objects are transparent. The scenes are in 2D with a 1D image. yellow circles are point lights; the ray tracer supports shadows. Draw all of the rays that would be cast while raytracing each scene. Use a maximum recursion depth of 3. (Don't worry about precisely what counts as depth 3 or depth 4; I just care that recursion is being performed correctly when necessary and that important rays are not missing. There are no more than 20 rays in the "exact" solution.)

Problem 11 (5 points)

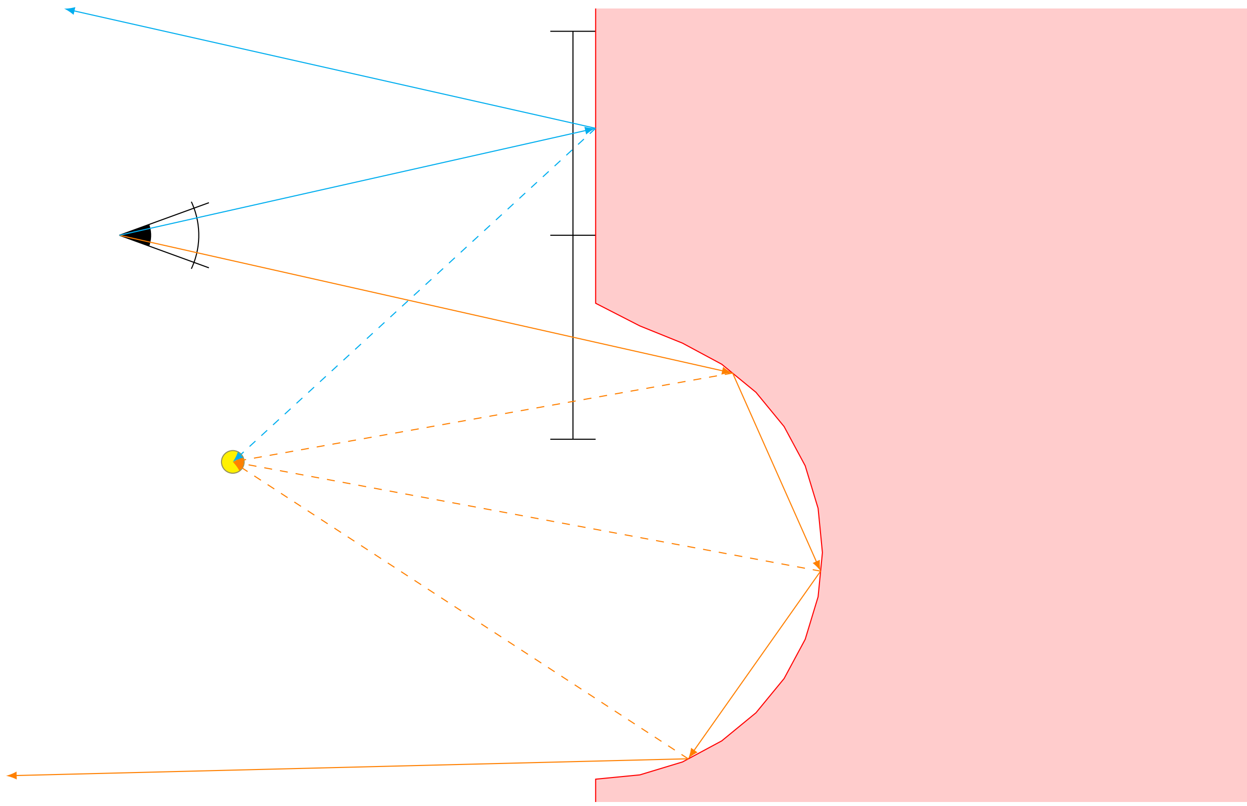


Problem 12 (2 points)

Explain why (a) a flashlight shined on a nearby wall is much brighter than when shined on a far wall but (b) a laser pointer appears about the same brightness in both cases.

(a) A flashlight falls off with distance since the light rays spread out; the same amount of light energy illuminates a larger wall area. (b) Laser pointers do not spread out with distance, so the light does not fall off much. (A very small amount of light energy is lost through interactions with particles in the air.)

Problem 13 (5 points)

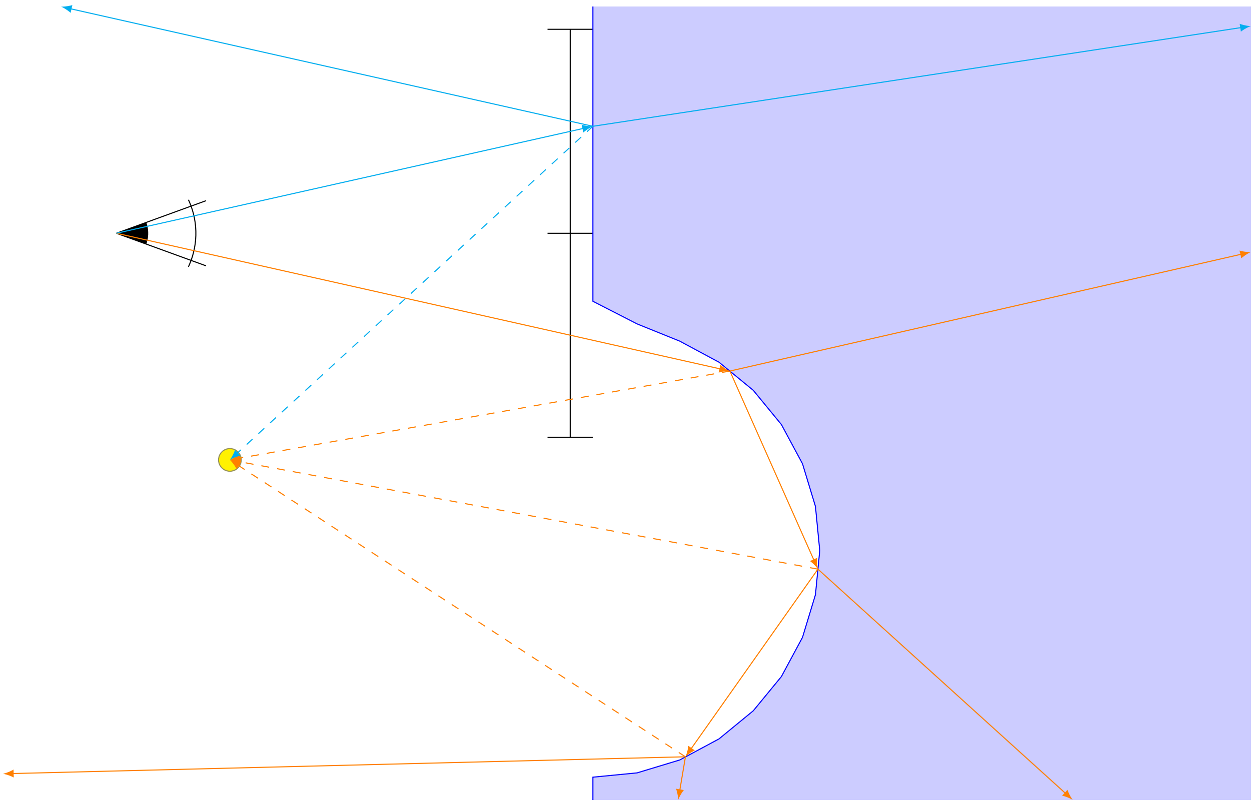


Problem 14 (3 points)

Write a routine that rasterizes a filled circle. That is, it draws pixels that lie on or inside the circle but not pixels that lie outside it. Your routine should be written in C++-like syntax and use the signature `void fill_circle(int x, int y, int r);`, where (x, y) and r are the circle's center and radius. You may call the routine `void draw(int x,int y);` to set the pixel (x, y). You may assume that the circle lies entirely inside the image.

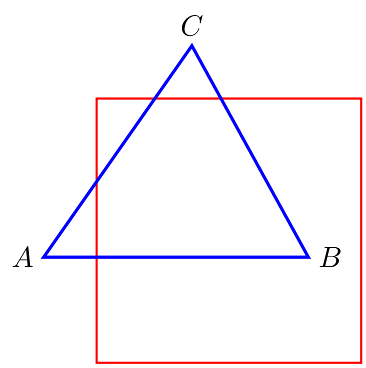
```
void fill_circle(int x, int y, int r)
{
  for(int i=-r;i<=r;i++)
    for(int j=-r;j<=r;j++)
      if(i*i+j*j<=r*r)
        draw(x+i,y+j);
}
```

Problem 15 (5 points)



Problem 16 (3 points)

Apply one of the clipping algorithms that we learned in class, step by step, to the triangle ABC. The clipping region is the red square. *Note that points are being awarded for demonstrating the steps of the algorithm. Merely showing what the results might look like does not score points.*



A correct solution for the triangle-based algorithm would clip the triangle against the edges of the square one by one. Triangles that get clipped into a quad should be divided back into two triangles. A correct solution for the polygon-based algorithm would maintain a polygon (initially the triangle) and clip it against the edges of the square one by one.

[1]

[1] Total points: 50