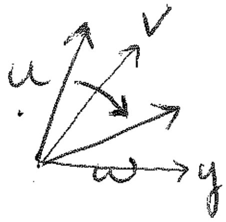
want to show $R^T R = I$ or $R R^T = I$

or $R^T = R^{-1}$



$w = Ru$

$y = Rv$

$\|w\| = \|u\|$

$\|y\| = \|v\|$

$w \cdot y = \|w\| \, \|y\| \cos\theta$

$u \cdot v = \|u\| \, \|v\| \cos\theta$

$w \cdot y = u \cdot v$

$u^T v = w^T y = (Ru)^T Rv$

$= u^T (R^T R) v$

$u^T \underbrace{(I - R^T R)}_{A} v = 0$

$u^T A v = 0$ any u, v

$= A_{ik}$ $\quad u = e_i = \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix} \leftarrow i$

$v = e_k$

$A = 0$

$\Downarrow$

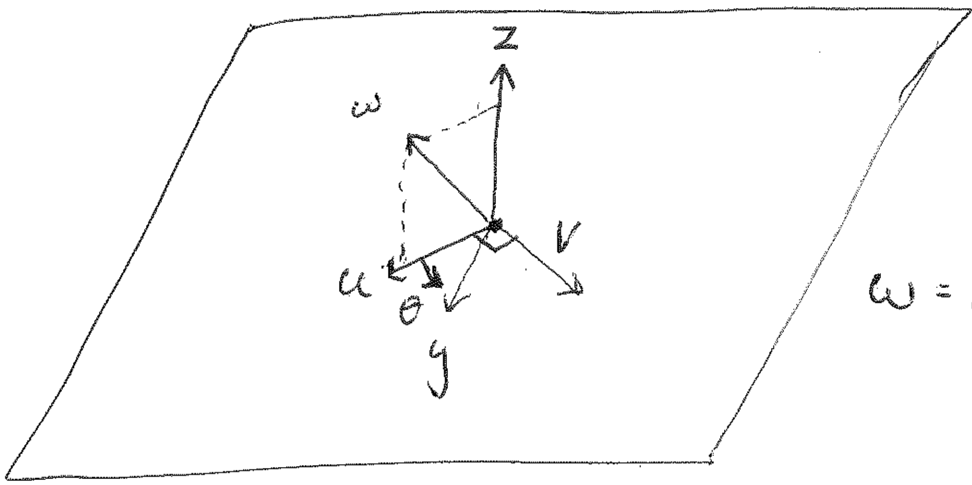
$\boxed{R^T R = I}$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{reflection}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}^T \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

rotation: $\det(R) = 1$

R rotate around $\vec{z}$ by θ. $\|\vec{z}\| = 1$



$Rz = z$

$w = a\vec{z} + \vec{u}$

$\vec{z} \cdot \vec{u} = 0$

$y = Ru$

$= bu + cv$

$v = z \times u$

$= z \times w - \underbrace{(z \times z)}_{0}(w \cdot z)$

$= z \times w$

$\|v\| = \|z\| \, \|u\| \sin \underset{\frac{\pi}{2}}{\phi} = \|u\|$

$w' = Rw = Raz + Ru$

$= az + \underbrace{Ru}_{y}$

$w \cdot z = a \underbrace{z \cdot z}_{1} + \underbrace{u \cdot z}_{0}$

$a = w \cdot z$

$w = (w \cdot z) z + u$

$u = w - (w \cdot z) z$

$\|y\| = \|u\|$

$y \cdot u = \|y\| \, \|u\| \cos\theta$

$b\,(u \cdot u) + c\underbrace{(v \cdot u)}_{0} = (u \cdot u) \cos\theta$

$b = \cos\theta$

$u \cdot u = y \cdot y = b^2 (u \cdot u) + 2bc \underbrace{(u \cdot v)}_{0} + c^2 (v \cdot v)$

$(1 - b^2)\, u \cdot u = c^2 (v \cdot v)$

$\sin^2\theta \; (u \cdot u) = c^2 (v \cdot v) = c^2 (u \cdot u) \Rightarrow c = \pm \sin\theta$ want $(+)$

$R\omega =$

$$\omega' = az + Ru$$

$$= (z\cdot\omega)z + y$$

$$= (z\cdot\omega)z + bu + cv$$

$$= (z\cdot\omega)z + (\cos\theta)(\omega - (z\cdot\omega)z) + (\sin\theta)(z\times\omega)$$

$$\boxed{R = zz^T + (\cos\theta)(I - zz^T) + (\sin\theta)\,z^*}$$

$$\boxed{z^*}\omega = z\times\omega$$