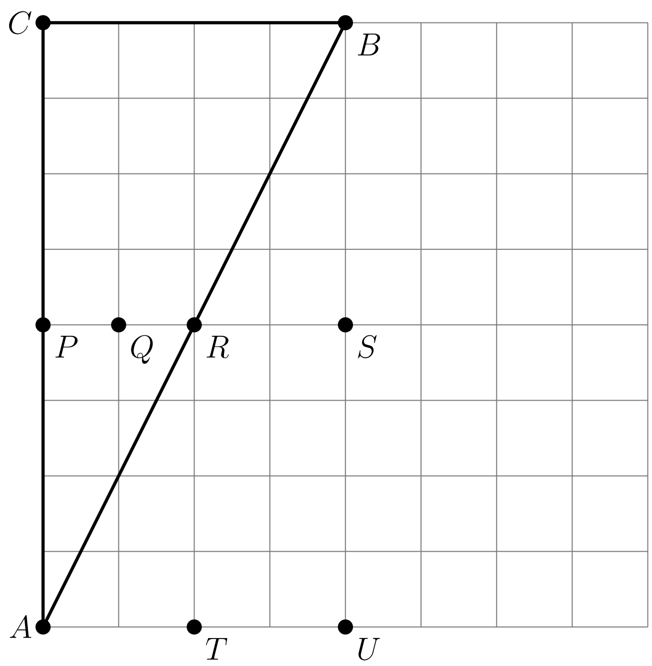
CS 130, Midterm

Solutions

Read the entire exam before beginning. **Manage your time carefully.** This exam has 32 points; you need 27 to get full credit. Additional points are extra credit. 32 points → 1.6 min/point. 27 points → 1.9 min/point.

Problem 1 (3 points)

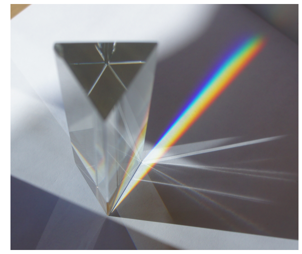
Compute the barycentric weights for each of the points below. You must put your results in the table for them to count. You do not need to show your work.



Point	α	β	γ
A	1	0	0
B	0	1	0
C	0	0	1
P	$\frac{1}{2}$	0	$\frac{1}{2}$
Q	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
R	$\frac{1}{2}$	$\frac{1}{2}$	0
S	$\frac{1}{2}$	1	$-\frac{1}{2}$
T	1	$\frac{1}{2}$	$-\frac{1}{2}$
U	1	1	-1

Problem 2 (2 points)

A prism is a transparent object that has different indexes of refraction for different wavelengths of light. As a result, for example, red and yellow light bend by different angles as they pass through the prism. In this way, sunlight is divided into a rainbow upon passing through a prism. (See the photograph to the right.) What would be observed if light from a computer screen displaying a solid white background were passed through the prism instead of white light from the sun?



Instead of a rainbow, you might see colored bands for red, green, and blue (in the same places that they occur in the rainbow) with no light between those bands. If the red, green, and blue are not pure, a few bands are observed. A rainbow will not be observed. (For example, sunlight contains light that is actually yellow. Yellow in a computer monitor is red plus green.)

Problem 3 (2 points)

Explain why (a) a flashlight shined on a nearby wall is much brighter than when shined on a far wall but (b) a laser pointer appears about the same brightness in both cases.

(a) A flashlight falls off with distance since the light rays spread out; the same amount of light energy illuminates a larger wall area. (b) Laser pointers do not spread out with distance, so the light does not fall off much. (A very small amount of light energy is lost through interactions with particles in the air.)

Problem 4 (2 points)

A plane (in 3D) can be defined in terms of a point $\mathbf{p}$ on the plane and the normal $\mathbf{n}$ to that plane. You are also given a second plane containing the point $\mathbf{r}$ and having normal direction $\mathbf{m}$. Find the *direction* of the line where the two planes intersect.

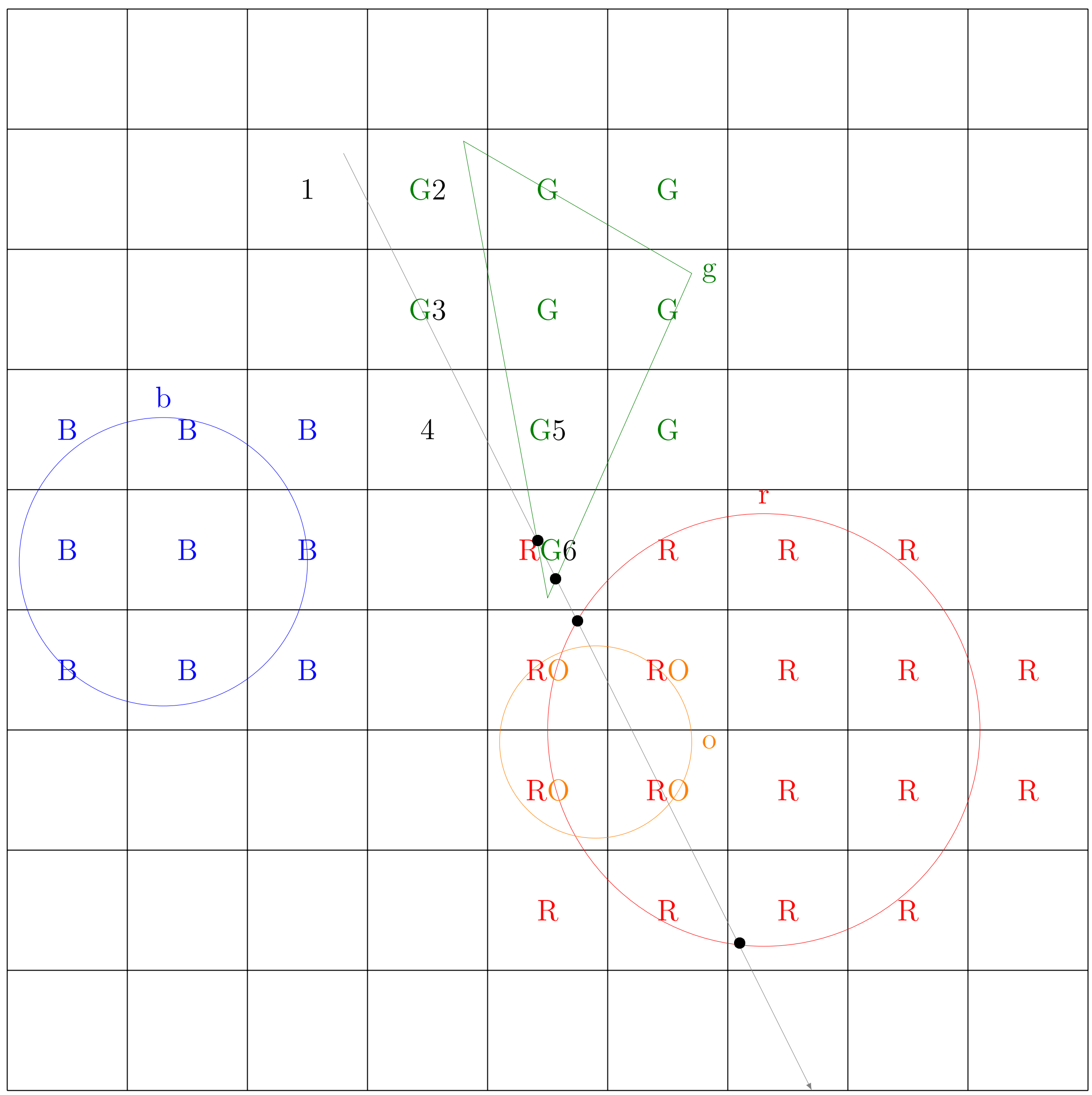
The direction of the line must be orthogonal to the normals of both planes, so that

$$\mathbf{u} = \frac{\mathbf{n} \times \mathbf{m}}{\|\mathbf{n} \times \mathbf{m}\|}.$$

Problem 5 (3 points)

Below is a raytracing acceleration structure. Label the following:

1. Label grid cells with "R", "G", "B", or "O" to indicate that a pointer to the Red, Green, Blue, or Orange object will be stored there. (The objects have been labeled "r", "g", "b", and "o" in case you are unable to see the colors.)
2. Number cells ("1", "2", "3", ...) in the order they will be visited to test for intersections along the gray ray. Cells that should not be visited should not be numbered.
3. Place a "•" at each intersection point that will be computed. Intersections that should not be computed should not be marked.



Problem 6 (3 points)

The equation $2z = x^2 + y^2$ represents a paraboloid. Compute *all* of the intersection *locations* between this paraboloid and the ray with endpoint $\mathbf{e} = \langle 0, -1, 3 \rangle$ and direction $\mathbf{u} = \langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \rangle$.

$$\begin{aligned}
x &= \frac{t}{3} \\
y &= \frac{-3 - 2t}{3} \\
z &= \frac{9 + 2t}{3} \\
2z &= x^2 + y^2 \\
2\left(\frac{9 + 2t}{3}\right) &= \left(\frac{t}{3}\right)^2 + \left(\frac{-3 - 2t}{3}\right)^2 \\
6(9 + 2t) &= t^2 + (3 + 2t)^2 \\
0 &= 5t^2 - 45 \\
t^2 &= 9 \\
t &= \pm 3
\end{aligned}$$

The negative solution does not lie on the ray, so $t = 3$. Plugging this in we get the intersection location $(1, -3, 5)$.

Problem 7 (3 points)

The equation $2z = x^2 + y^2$ represents a paraboloid. Given a point (x, y, z) that is on the paraboloid, compute the normal direction at that location.

This can be done easily as a parametric equation: $(u, v, (x^2 + y^2)/2)$. It can also be done as an implicit equation: $f(x, y, z) = x^2 + y^2 - 2z = 0$. The latter approach is simpler, so we will do it that way.

$$\nabla f = \begin{pmatrix} 2x \\ 2y \\ -2 \end{pmatrix}$$

$$\mathbf{n} = \frac{\nabla f}{\|\nabla f\|} = \frac{1}{\sqrt{x^2 + y^2 + 1}} \begin{pmatrix} x \\ y \\ -1 \end{pmatrix}$$

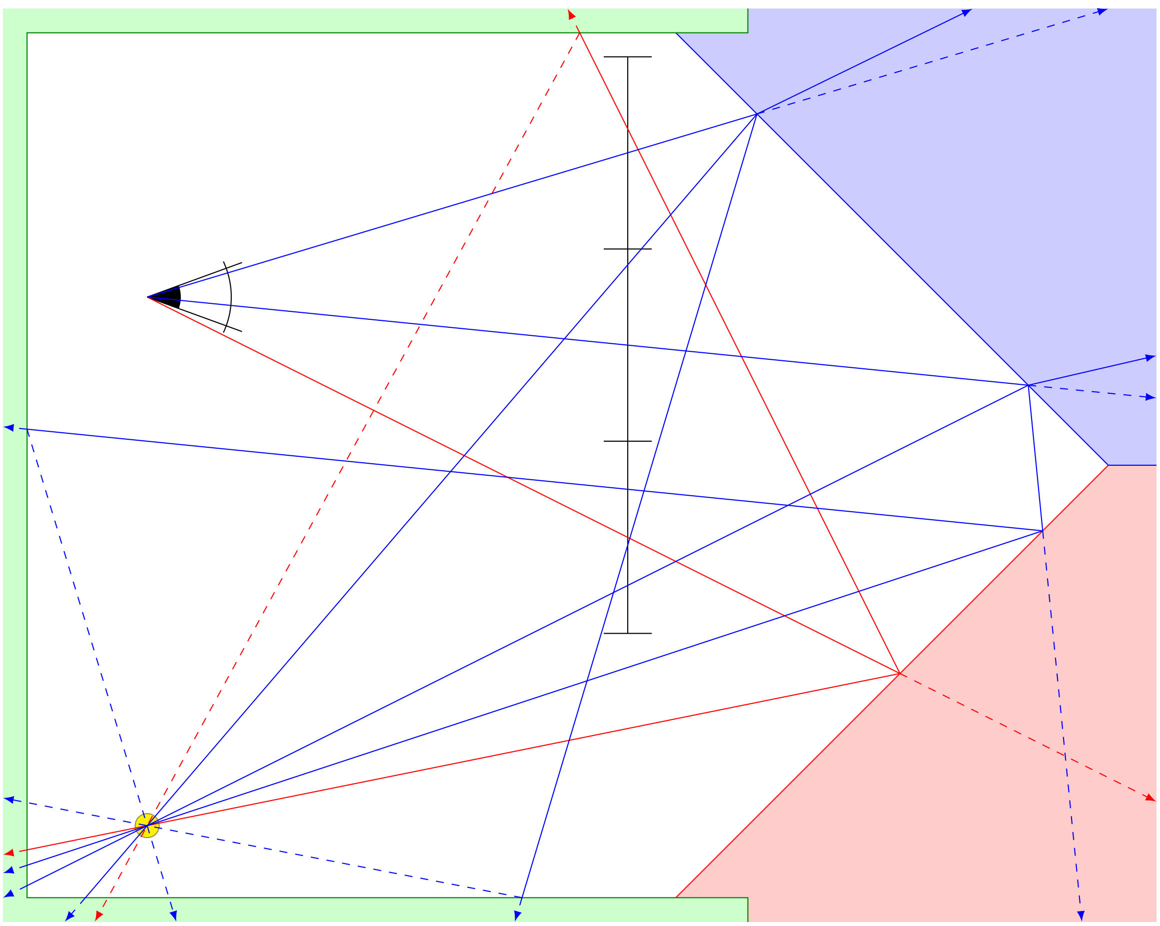
Problem 8 (3 points)

Construct a transformation that maps numbers x in the interval $[a, b]$ to numbers y in the interval $[c, d]$. (All points x must map to a valid y, and all values y must be achieved by some value of x.)

$$
\begin{aligned}
y &= rx + s \\
c &= ra + s \\
d &= rb + s \\
d - c &= r(b - a) \\
r &= \frac{d - c}{b - a} \\
s &= c - ra = c - \frac{d - c}{b - a}a \\
y &= rx + s \\
y &= \frac{d - c}{b - a}x + c - \frac{d - c}{b - a}a \\
y &= \frac{d - c}{b - a}(x - a) + c
\end{aligned}
$$

In the raytracing problems below, green objects are wood, red objects are reflective, and blue objects are transparent. The scenes are in 2D with a 1D image. yellow circles are point lights; the ray tracer supports shadows. Draw all of the rays that would be cast while raytracing each scene. Use a maximum recursion depth of 6. (Don't worry about precisely what counts as depth 6; I just care that recursion is being performed correctly when necessary and that important rays are not missing. There are no more than 25 rays in the "exact" solution.)

Problem 9 (6 points)



Problem 10 (3 points)

For some of these questions, you will be asked to make calculations without using a calculator. You are provided with a trig table on the right, which is in degrees. You will get credit for your calculations as long as you are within 10% of the correct value. Snell's law $\mathbf{n}_i \sin\theta_i = \mathbf{n}_o \sin\theta_o$ relates the incoming angle θ_i and outgoing angle θ_o for rays passing from one medium to another (e.g., from air to glass). We will assume that our materials have $\mathbf{n}_i = 1.5$ and $\mathbf{n}_o = 1$. For each of the following incoming angles, estimate the angle of the **reflected ray** and the **transmitted ray**.

(a) $\theta_i = 10°$
(b) $\theta_i = 30°$
(c) $\theta_i = 50°$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0.00	1.00	0.00
10	0.17	0.98	0.18
20	0.34	0.94	0.36
30	0.50	0.87	0.58
40	0.64	0.77	0.84
50	0.77	0.64	1.19
60	0.87	0.50	1.73
70	0.94	0.34	2.75
80	0.98	0.17	5.67
90	1.00	0.00	∞

(a) reflected is 10°, transmitted is 15.1°.
(b) reflected is 30°, transmitted is 48.6°.
(c) reflected is 50°, and no transmitted ray exists. Complete internal reflection.

Problem 11 (2 points)

Plants absorb a portion of the sunlight that they receive and use it to produce sugars in a process called photosynthesis. What colors of light do you think the plants are using?

Plants appear green because they reflect the green light that they receive. They absorb the colors that are not green. They use red/orange light and blue/violet light for photosynthesis. They do not use green or yellow.

[1]

[1] Total points: 32