CS 130, Midterm

Solutions

1	2	3	4	5	6	7	8	9	10	11	Σ

Read the entire exam before beginning. **Manage your time carefully.** This exam has 33 points; you need 25 to get full credit. Additional points are extra credit. 33 points → 2.4 min/point. 25 points → 3.2 min/point. *You are not allowed to ask questions during this exam; do your best with the information provided.*

Problem 1 (2 points)

What change would need to be made to your raytracer to implement antialiasing?

The ray tracer should be changed to cast multiple rays per pixel. This could be implemented reasonably in `Render_World::Render_Pixel` or `Render_World::Render`.

Problem 2 (2 points)

What are the barycentric coordinates of the center of an equilateral triangle?

$\alpha = \beta = \gamma = \frac{1}{3}$. They must be equal due to symmetry, and they must add to 1.

Problem 3 (2 points)

Why does the intensity of a point light fall off with distance?

Imagine that the light is illuminating a dome centered at the light. The same amount of energy is emitted by the light no matter how big the dome is, but for a bigger dome that energy is spread over a larger surface area. The farther surface thus experiences less intense light.

Problem 4 (2 points)

Why does a directional light not fall off with distance?

The light source for a directional light is assumed to be so far away that all light essentially comes from the same direction. Since the light is so far away, changes in brightness due to the location of an object would be negligible.

Problem 5 (2 points)

Given two vectors $\vec{u}$ and $\vec{w}$, how do we determine whether the vectors are orthogonal?

$\vec{u} \cdot \vec{w} = 0$

Problem 6 (3 points)

Given a large number of triangles, construct an efficient algorithm to identify all pairs of triangles that intersect. You may assume that you have a triangle-triangle intersection routine available. The use case here is that the triangles are rather *practically* distributed (for example, a triangulated representation of all of the objects in this classroom), the number of triangles is very large (perhaps a million), but the number of intersecting pairs is relatively small (perhaps only a hundred or so). Note that we are not after optimal or ever provable time bounds. We mostly just want something that is simple to implement and reasonably efficient in practice.

Use the grid-based acceleration structure with a list of objects in each grid cell. For each triangle add its pointer to the lists of the cells that it touches. Then, for each cell, intersect all pairs of triangles in the list for that cell. As long as the triangles are reasonably distributed and the grid cells are reasonably small, the per-cell lists will tend to be quite small. As such, the all-pairs test within each cell should not be very expensive.

Problem 7 (3 points)

A surface (rather like a spiral staircase) is given by the parametric equations $x = r\cos\theta$, $y = r\sin\theta$, $z = \theta$. What is the normal direction at an arbitrary point on the surface? You may express your result as a function of (r, θ) or (x, y, z) as you prefer.

$$\mathbf{w} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ \theta \end{pmatrix}$$

$$\mathbf{w}_r = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$\mathbf{w}_\theta = \begin{pmatrix} -r\sin\theta \\ r\cos\theta \\ 1 \end{pmatrix}$$

$$\mathbf{w}_r \times \mathbf{w}_\theta = \begin{pmatrix} (\sin\theta)1 - 0(r\cos\theta) \\ 0(-r\sin\theta) - (\cos\theta)1 \\ (\cos\theta)(r\cos\theta) - (\sin\theta)(-r\sin\theta) \end{pmatrix} = \begin{pmatrix} \sin\theta \\ -\cos\theta \\ r \end{pmatrix}$$

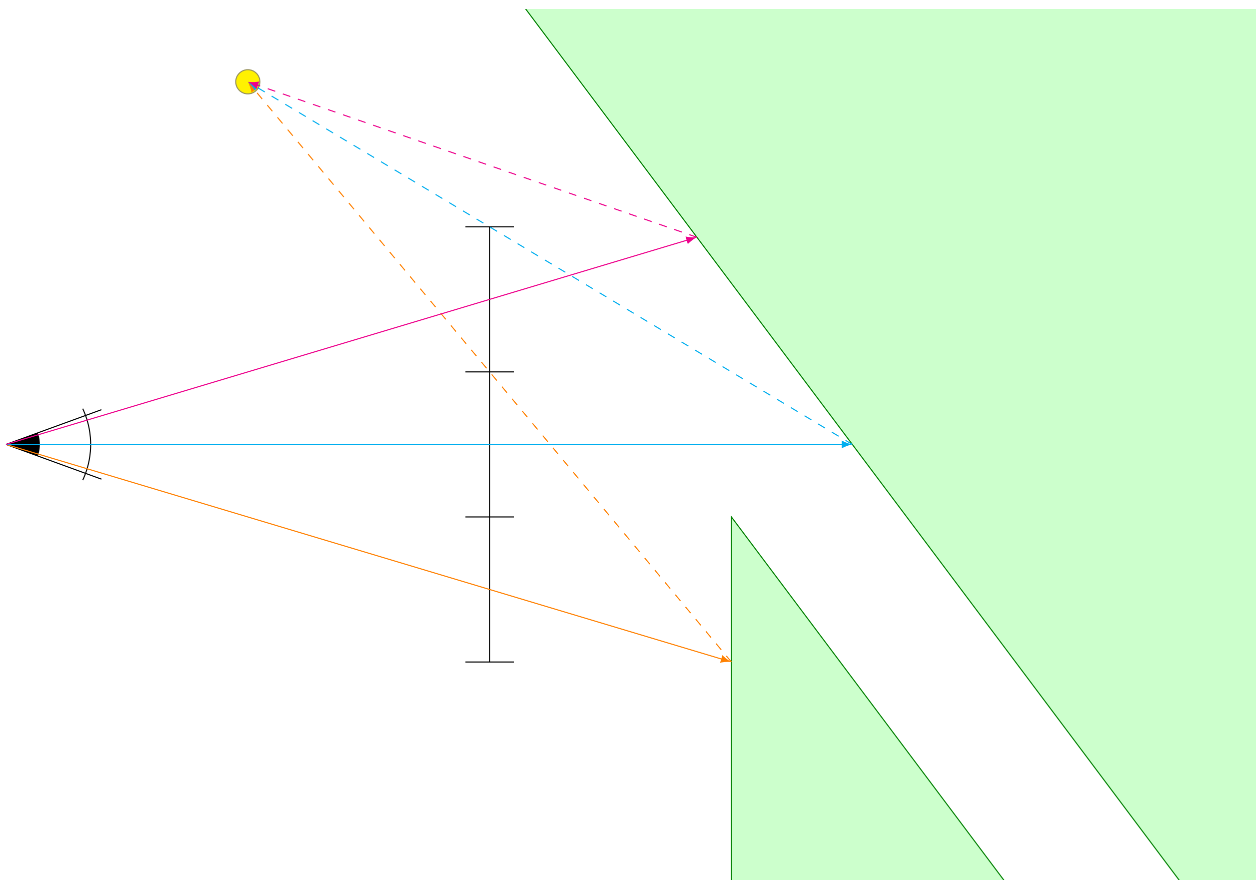
$$\|\mathbf{w}_r \times \mathbf{w}_\theta\|^2 = (\sin\theta)^2 + (-\cos\theta)^2 + r^2 = r^2 + 1$$

$$n = \frac{\mathbf{w}_r \times \mathbf{w}_\theta}{\|\mathbf{w}_r \times \mathbf{w}_\theta\|} = \frac{1}{\sqrt{r^2+1}} \begin{pmatrix} \sin\theta \\ -\cos\theta \\ r \end{pmatrix} = \frac{1}{\sqrt{x^2+y^2+1}\sqrt{x^2+y^2}} \begin{pmatrix} y \\ -x \\ x^2+y^2 \end{pmatrix}$$

In the raytracing problems below, green objects are wood, red objects are reflective, and blue objects are transparent. The scenes are in 2D with a 1D image. yellow circles are point lights; the ray tracer supports shadows. Draw all of the rays that would be cast while raytracing each scene. Use a maximum recursion depth of 6. (Don't worry about precisely what counts as depth 6; I just care that recursion is being performed correctly when necessary and that important rays are not missing. There are no more than 25 rays in the "exact" solution.)

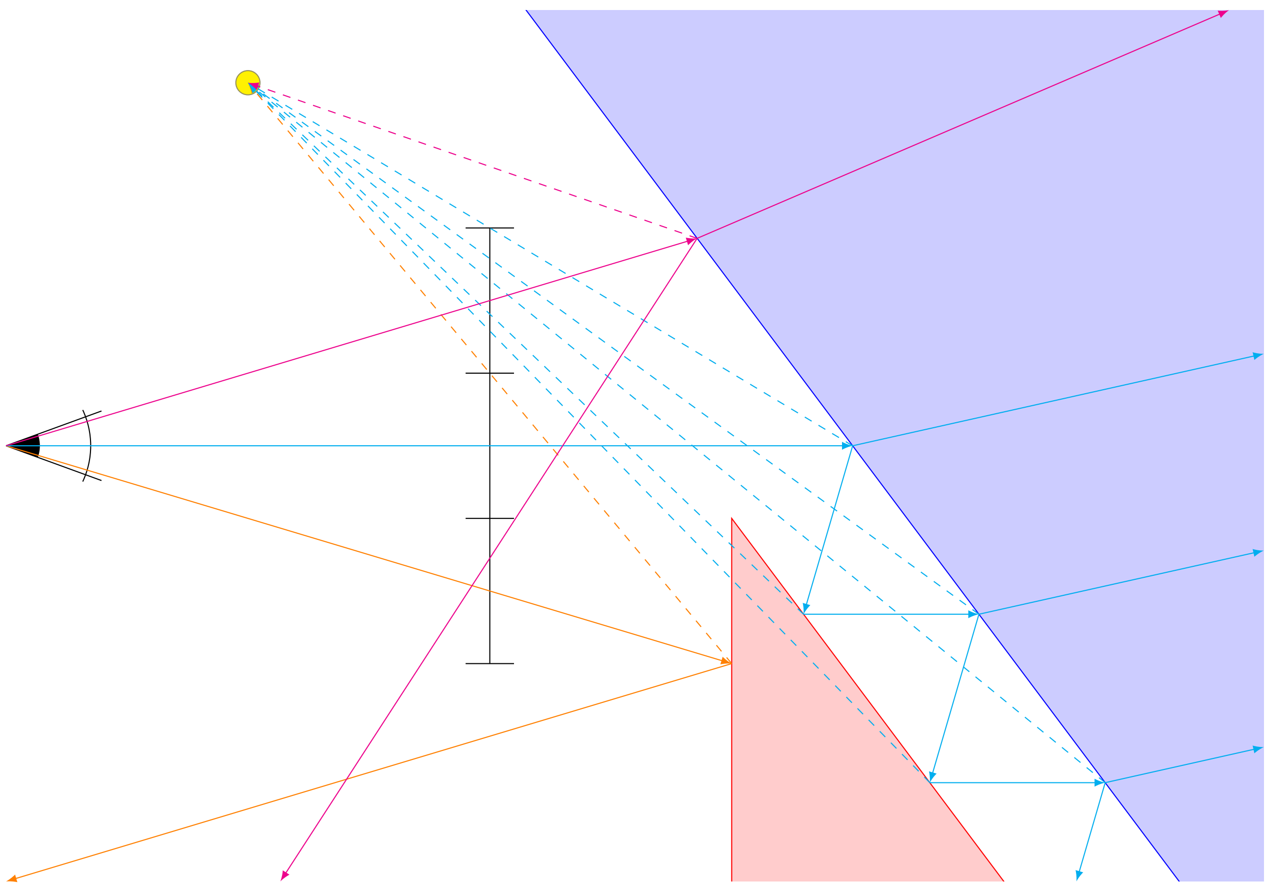
Problem 8 (6 points)

For this problem, assume a recursion depth of 6. (If you have recursion depth 5-7, it is fine.)



Problem 9 (6 points)

For this problem, assume a recursion depth of 6. (If you have recursion depth 5-7, it is fine.)



Problem 10 (2 points)

Why do we use red, green, and blue when producing images? Why not two or four colors instead of three? Why not a different set of three colors?

The human eye uses three types of color-sensitive cells to distinguish colors; these cells are most sensitive to red, green, and blue light. The eye thus perceives other colors as combinations of these colors.

Problem 11 (3 points)

For some of these questions, you will be asked to make calculations without using a calculator. You are provided with a trig table on the right, which is in degrees. You will get credit for your calculations as long as you are within 10% *of the correct value.* Snell's law $\mathbf{n}_i \sin\theta_i = \mathbf{n}_o \sin\theta_o$ relates the incoming angle θ_i and outgoing angle θ_o for rays passing from one medium to another (e.g., from air to glass). We will assume that our materials have $\mathbf{n}_i = 1.5$ and $\mathbf{n}_o = 1$. For each of the following incoming angles, estimate the angle of the **reflected ray** and the **transmitted ray**.

(a) $\theta_i = 10°$
(b) $\theta_i = 30°$
(c) $\theta_i = 50°$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0.00	1.00	0.00
10	0.17	0.98	0.18
20	0.34	0.94	0.36
30	0.50	0.87	0.58
40	0.64	0.77	0.84
50	0.77	0.64	1.19
60	0.87	0.50	1.73
70	0.94	0.34	2.75
80	0.98	0.17	5.67
90	1.00	0.00	∞

(a) reflected is 10°, transmitted is 15.1°.
(b) reflected is 30°, transmitted is 48.6°.
(c) reflected is 50°, and no transmitted ray exists. Complete internal reflection.

[1]

[1]Total points: 33