HW2 Solutions

Q1 [15 pts] P.79 Ex.2.5.2.

Answer:

a) ECLOSE(p) = {p,q,r}
ECLOSE(q) = {q}
ECLOSE(r) = {r}

b) Any string over {a,b,c} whose length is less than or equal to 3,
with the exception of {bba,bbb,bbc}.

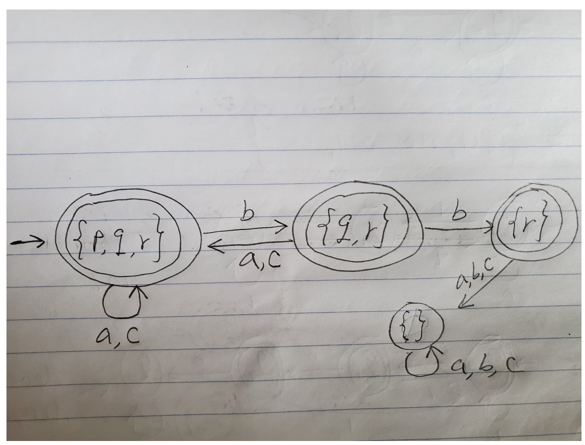
In other words, the following strings:
{epsilon, a, b, c,
aa, ab, ac, ba, bb, bc, ca, cb, cc,
aaa, aab, aac, aba, abb, abc, aca, acb, acc,
baa, bab, bac, bca, bcb, bcc,
caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc}

c) Starting from ECLOSE(p) = {p,q,r}, we define the following
transitions in the DFA:

transition ({p,q,r},a)={p,q,r}
transition ({p,q,r},b)={q,r}
transition ({p,q,r},c)={p,q,r}

Then, continuning with the state {q,r}, we define:

transition ({q,r},a)={p,q,r}
transition ({q,r},b)={r}
transition ({q,r},c)={p,q,r}



For the state {r}, we define:

transition ({r},a)=empty set
transition ({r},b)=empty set
transition ({r},c)=empty set

Finally, for the state empty (or {}), we define

transition ({},a)={}
transition ({},b)={}
transition ({},c)={}

The state state is {p,q,r} and the final states are {p,q,r}, {q,r} and {r}.

Q2 [10 pts]

Part a)

(0+1)*1(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)

Part b)

(0+10)*(e+1+11)(0+01)*

Note that other valid regex's may also exist.

Q3 [20 pts] Convert the following DFA to a regular expression by
following the state elimination technique. Show all the important
intermediate steps.

```
         || 0 | 1
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->*a     || b | c
    b    || a | d
    c    || d | a
   *d    || c | b
```

Answer: Please see the last page for details.
Note that here we may also convert the given the DFA to an
epsilon-NFA with a unique final state and then perform state
elimination.

Q4 [10 pts] P.108 Ex.3.2.6: c), d)

Answer:

c) The set of prefixes of strings in L.

d) The set of all substrings of L (including epsilon).

Q5 [20 pts] P.121-122 Ex.3.4.1: e), g)

Answer:

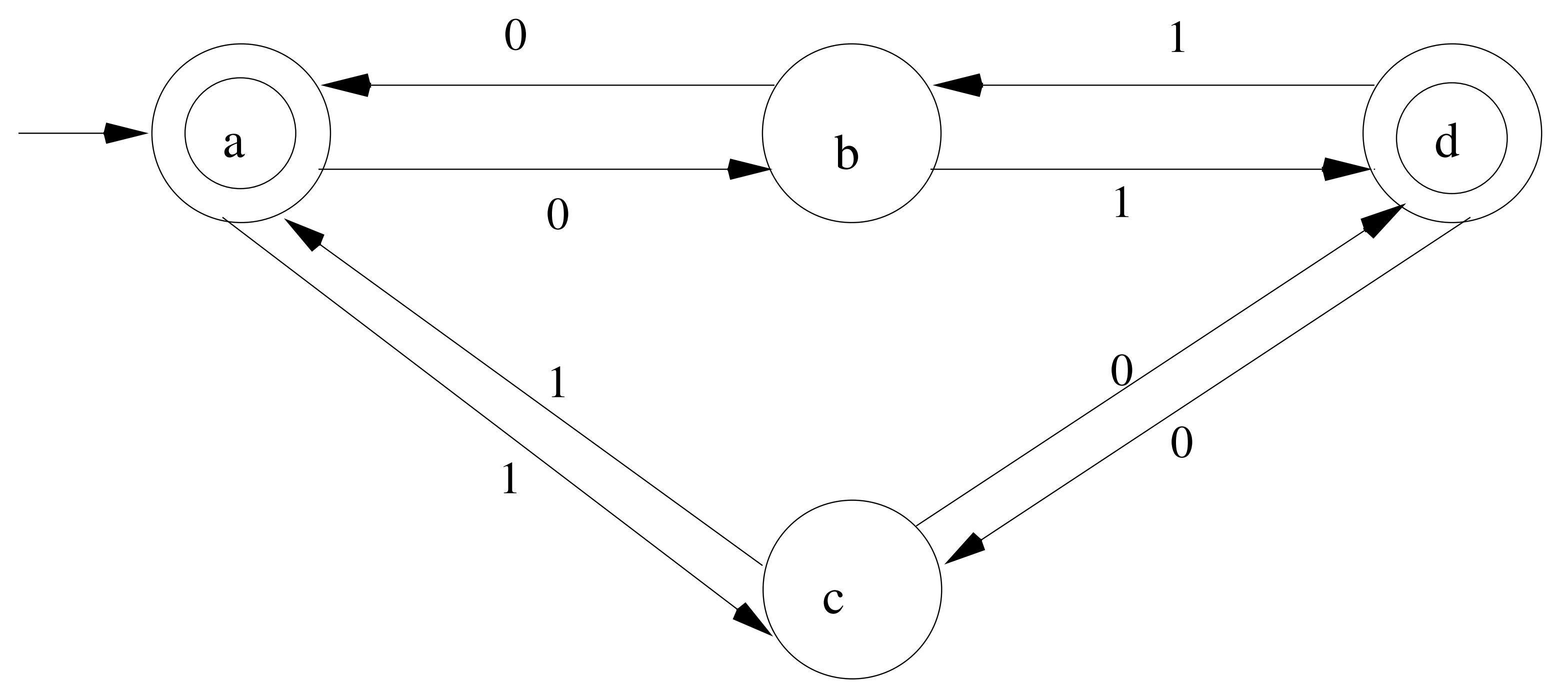
e)

Replace R by symbol a, S by b and T by c. The lefthand side becomes
(a+b)c. The righthand side is ac+bc. L((a+b)c) = L(a+b)L(c) = {a,b}{c}
= {ac,bc} = L(ac+bc).

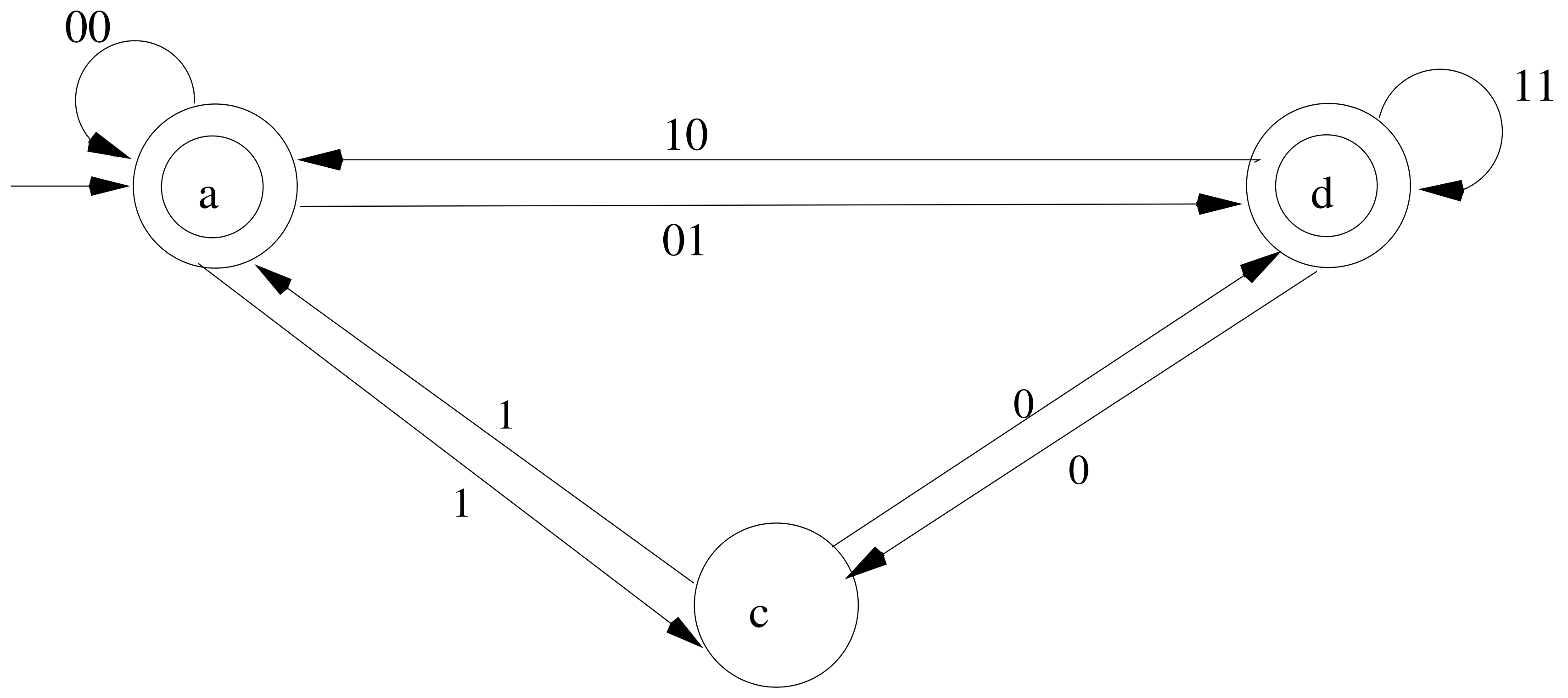
g)

Replace R by a. The lefthand side becomes (e+a)*.
The righthand side becomes a*, which represents all strings over
the unary alphabet {a} (i.e., its universe). Obviously, the LHS is
contained in the RHS. Since L(a) is contained in L(e+a),
L(a*) is contained in L((e+a)*). Hence, the RHS is contained in
the LHS as well, and both sides are equal.

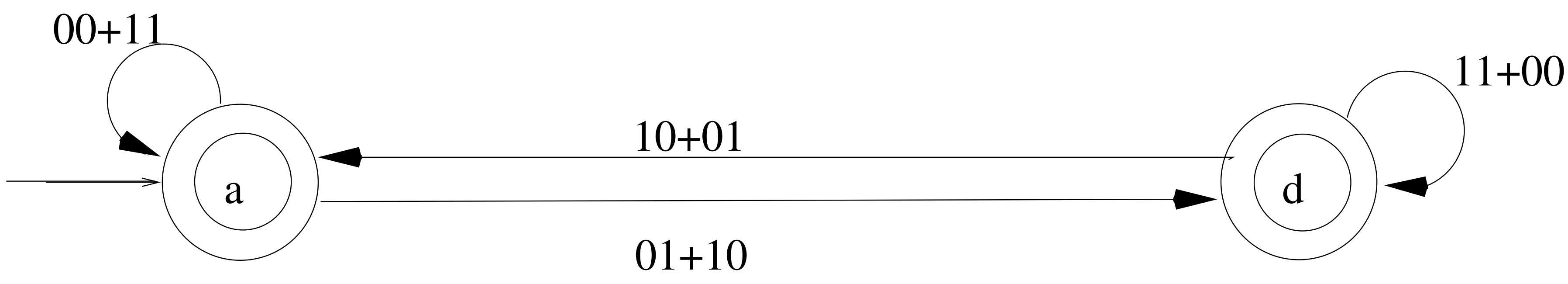
Solution for Q3:



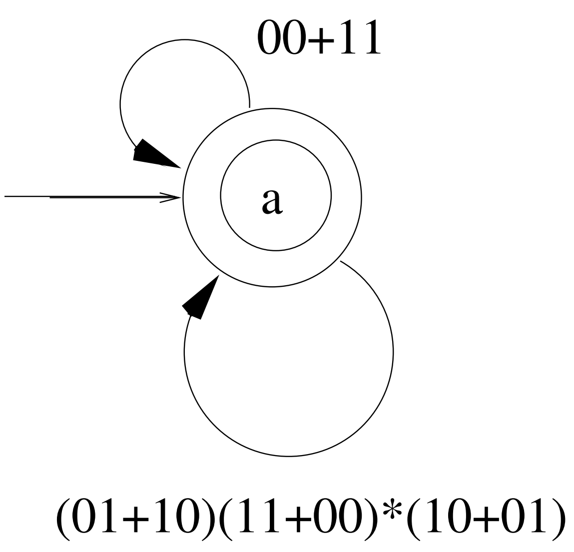
1) eliminate state (b)



2) eliminate state (c)

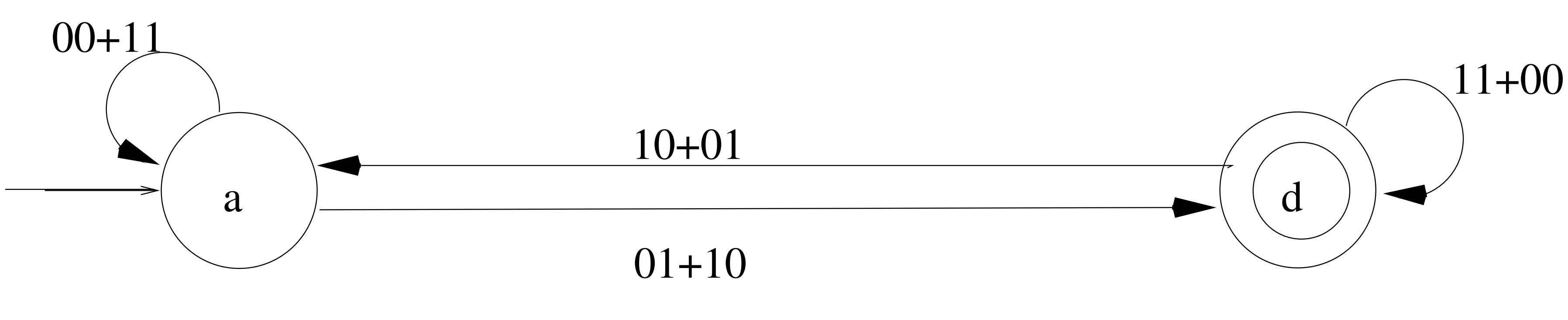


3) Regard a as the only final state and eliminate state d:



Hence, $R_1 = (00+11+(01+10)(11+00)^*(10+01))^*$

Regard d as the only final state:



Hence, $R_2 = (00+11+(01+10)(11+00)^*(10+01))^*(01+10)(00+11)^*$

4) final regular expression

$$R = R_1 + R_2$$