CS 150 (Closed-book) Midterm Test

Nov. 13, Wed., 6:30-7:50pm, 2024

Total: 75 points

Name:

UCR Net ID:

QUESTION 1. [10 pts] Design a DFA to accept the following language:

$$L = \{x \mid x \in \{0,1\}^*, x \text{ contains at least one 0 and at least two } 1'\text{s}\}$$

Answer:

In the following DFA, the subscripts a, b in state $q_{a,b}$ count the numbers of 0's and 1's in the input string, respectively.

	0	1
$\rightarrow q_{0,0}$	$q_{1,0}$	$q_{0,1}$
$q_{1,0}$	$q_{1,0}$	$q_{1,1}$
$q_{0,1}$	$q_{1,1}$	$q_{0,2}$
$q_{1,1}$	$q_{1,1}$	$q_{1,2}$
$q_{0,2}$	$q_{1,2}$	$q_{0,2}$
$*q_{1,2}$	$q_{1,2}$	$q_{1,2}$

Either a state diagram or table would be acceptable. Give partial credits for DFAs with some relevant properties.

QUESTION 2. [10 pts] Convert the following NFA to a DFA:

	0	1
$\rightarrow q_0$	$\{q_1\}$	$\{q_0, q_2\}$
$*q_1$	$\{q_0, q_2\}$	$\{q_2\}$
q_2	$\{q_1, q_2\}$	$\emptyset$

Answer:

	0	1
$\rightarrow \{q_0\}$	$\{q_1\}$	$\{q_0, q_2\}$
$*\{q_1\}$	$\{q_0, q_2\}$	$\{q_2\}$
$\{q_2\}$	$\{q_1, q_2\}$	$\emptyset$
$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_2\}$
$*\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_2\}$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
$\emptyset$	$\emptyset$	$\emptyset$
$*\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$

It's okay to exclude/include the inaccessible state $\{q_0, q_1\}$. Give partial credits for correct steps. Deduct 1 pt for missing the state $\emptyset$.

QUESTION 3. [10 pts] Prove or disprove the following identities. Note that you can disprove an identiy by means of a counterexample.

1. $(1^*0^*1^*)^* = (101)^* + 1^*0^*1^*$
2. $1^*(0+1)^* = 0^*(0+1)^*$

Answer:

1. False (2 pts). 010 is in the LHS but not the RHS (2 pts).
2. True (2 pts). We prove both sides are equal to $(0+1)^*$. Clearly, $L(1^*(0+1)^*) \subseteq L((0+1)^*)$ because the latter represents all binary strings (*i.e.*, it is the universe). (2 pts) Since $\epsilon \in L(1^*)$, $L((0+1)^*) = L(\epsilon(0+1)^*) \subseteq L(1^*(0+1)^*)$. (2 pts) Hence, $L(1^*(0+1)^*)$ $= L((0+1)^*)$. Similarly, we can prove $L(0^*(0+1)^*) = L((0+1)^*)$.

QUESTION 4. [10 pts] Give a regular expression for the following language:

$$L = \{x \mid x \in \{0,1\}^*,\ x \text{ does not begin with } 11\}$$

Answer:

$$\epsilon + 0(0+1)^* + 1 + 10(0+1)^*$$

The regex is not unique. Give partial credits for regex's containing some correct components.

QUESTION 5.

1. [10 pts] Convert the following DFA to a regular expression by using the state elimination algorithm:

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

2. [5 pts] What is the language accepted by the above DFA? You may describe the language by giving the (mathematical) property of its strings.

Answer:

1. After eliminating state q_2, the arc label from q_1 to q_1 becomes 01^*0. (5 pts)
   After eliminating state q_1, the arc label from q_0 to q_0 becomes $0+1(01^*0)^*1$, which leaves the final answer as $(0+1(01^*0)^*1)^*$. (5 pts)
2. Binary numbers divisible by 3.

   Give partial credits for correct steps. If state q_1 is eliminated first, the final answer will be $(0+11+10(00+1)^*01)^*$.

QUESTION 6. [10 pts] Prove that the following language is not regular using the Pumping Lemma:

$$L = \{0^{i+j}1^i2^j \mid i, j \geq 0\}$$

Answer:

Let n be the constant in the Pumping Lemma. Pick $w = 0^{2n}1^n2^n \in L$. Let $w = xyz$ be any partition satisfying (i) $|y| > 0$ and $|xy| \leq n$. Clearly, $x = 0^i$ and $y = 0^j$ for some $i \geq 0$ and $j > 0$. Then $xyyz = 0^{2n+j}1^n2^n \notin L$, and thus a contradiction.

Give partial credits for correct/reasonable steps.

QUESTION 7. [10 pts] Convert the following DFA to the minimum-state equivalent DFA step-by-step using the TF algorithm.

$\rightarrow *q_0$	q_1	q_2
$*q_1$	q_0	q_2
q_2	q_3	q_0
q_3	q_2	q_4
q_4	q_2	q_5
q_5	q_2	q_3

Answer:

The DFA in Q5. Must show the state equivalence table first.

Give partial credits for correct steps. For tables mostly correct but with a few errors, deduct 1 or 2 pts for each error.