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Problem 1: Let $X = \{a, b, c\}$, $Y = \{a, d, e, f\}$, and $Z = \{f, g\}$. List all elements of the following sets:

$$X \cup Y \cup Z = \{a, b, c, d, e, f, g\}$$

$$X \cap Y = \{a\}$$

$$(Y - X) - Z = \{d, e\}$$

$$X \times X = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$$

$$\mathbf{P}(Z) = \{\emptyset, \{f\}, \{g\}, \{f, g\}\}$$

Note: $\mathbf{P}(Z)$ denotes the power set of Z.

Problem 2: Let $f, g : R \to R$ be the functions given by $f(x) = 2x - 3$, and $g(x) = x^3 + 1$. Give the formulas for the following functions:

$$f \circ g(x) = 2(x^3 + 1) - 3$$

$$g \circ f(x) = (2x - 3)^3 + 1$$

$$g \circ g(x) = (x^3 + 1)^3 + 1$$

$$f^{-1}(x) = \frac{(x + 3)}{2}$$

$$g^{-1}(x) = (x - 1)^{\frac{1}{3}}$$

Note: $f \circ g$ is the composition of g and f, that is $f \circ g(x) = f(g(x))$, and f^{-1} is the inverse of f.

Problem 3: Let X be a set of 10 distinct items. Give formulas for the following quantities (you do not have to compute the value.)

(a) What is the total number of subsets of X?

2^{10}

(b) In how many ways we can choose 6 items from X if the items in the choices are ordered and repetition is not allowed?

$P(10, 6) = \frac{10!}{4!}$

(c) In how many ways we can choose 6 items from X if the items in the choices are ordered and repetition is allowed?

$P_{rep}(10, 6) = 10^6$

(d) In how many ways we can choose 6 items from X if the items in the choices are not ordered and repetition is not allowed?

$C(10, 6) = \frac{10!}{6!4!}$

(e) In how many ways we can order X?

$P(10, 10) = 10!$