

NAME:

SID:

Problem 1: Determine the numerical values of the expressions below:

$$1 + 2 + \dots + 100 =$$

$$\gcd(198, 242) =$$

$$163 \text{ rem } 15 =$$

$$\binom{15}{4} =$$

$$\sum_{i=0}^{\infty} (1/3)^i =$$

Reminders:

- $\gcd(a, b)$ is the greatest common divisor of a and b
- $a \text{ rem } b$ is the remainder of a modulo b (often also denoted $a \bmod b$)

Problem 2: (10 points). Let X and Y be two finite sets with cardinalities $|X| = n$ and $|Y| = m$. Complete the following sentences.

(a) X has subsets.

(b) $X \times Y$ has elements.

(c) The number of permutations of Y is

(d) There are length- k sequences of elements from Y (with repetitions allowed).

(e) X has k -element subsets (for $0 \leq k \leq n$).

Problem 3: For each of the statements below, tell whether it is true or false.

Note: to discourage guessing, the answers will be graded as follows: correct = +2, no answer = 0, incorrect = -1.

statement	T/F
$\exists x \in \mathbb{R} : x^2 + x = 2$	
$\exists x \in \mathbb{R} : x^2 + x = -2$	
$\forall x \in \mathbb{R} : (x^2 > 4) \implies (x > 2)$	
$\forall x \in \mathbb{R} \exists y \in \mathbb{R} : xy^2 + x = 1$	
$\exists x \in \mathbb{R} \forall y \in \mathbb{R} : xy^2 + 2^x = 1$	

Reminders:

- \mathbb{R} denotes the set of real numbers.
- \forall denotes the universal quantifier (“for all”) and \exists denotes the existential quantifier (“there exists”).