NAME: SID:

Problem 1: Determine the numerical values of the expressions below:

$$
\begin{aligned}
1 + 2 + ... + 100 &= 5050 \\
\gcd(198, 242) &= 22 \\
163 \operatorname{rem} 15 &= 13 \\
\binom{15}{4} &= 1365 \\
\sum_{i=0}^{\infty} (1/3)^i &= \frac{3}{2}
\end{aligned}
$$

Reminders:

- $\gcd(a, b)$ is the greatest common divisor of a and b
- $a \operatorname{rem} b$ is the remainder of a modulo b (often also denoted a mod b)

Problem 2: (10 points). Let X and Y be two finite sets with cardinalities $|X| = n$ and $|Y| = m$. Complete the following sentences.

(a) X has 2^n subsets.

(b) $X \times Y$ has $n \cdot m$ elements.

(c) The number of permutations of Y is $m!$.

(d) There are m^k length-k sequences of elements from Y (with repetitions allowed).

(e) X has $\binom{n}{k}$ k-element subsets (for $0 \le k \le n$).

Problem 3: For each of the statements below, tell whether it is true or false.

Note: to discourage guessing, the answers will be graded as follows: correct = +2, no answer = 0, incorrect = -1.

statement	T/F
$\exists\, x \in \mathbb{R} \,:\, x^2 + x = 2$	T
$\exists\, x \in \mathbb{R} \,:\, x^2 + x = -2$	F
$\forall\, x \in \mathbb{R} \,:\, (x^2 > 4) \implies (x > 2)$	F
$\forall\, x \in \mathbb{R}\ \exists\, y \in \mathbb{R} \,:\, xy^2 + x = 1$	F
$\exists\, x \in \mathbb{R}\ \forall\, y \in \mathbb{R} \,:\, xy^2 + 2^x = 1$	T

Reminders:

- $\mathbb{R}$ denotes the set of real numbers.
- $\forall$ denotes the universal quantifier ("for all") and $\exists$ denotes the existential quantifier ("there exists").