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Problem 1: Let $X = \{a, b, c, d, e, f\}$ and $Y = \{a, e, h\}$.

(a) The power set of Y is $\mathcal{P}(Y) =$

(b) The union of X and Y is $X \cup Y =$

(c) The number of four-element subsets of X is

(d) The number of ways to order all elements of X is

(e) The number of functions that map Y into X is

Note: In questions (c), (d), (e) you need to first give the formula and then compute the numerical value.

Problem 2: For each of the statements below, determine whether it is true or false. Give a brief justification of your answer (at most 10 words). *Note:* to discourage guessing, incorrect T/F answers will receive negative credit.

statement	T/F	justification
$\exists\, x \in \mathbb{Z} \,:\, x^3 + 4x^2 - 2x + 3 = 0$		
$\exists\, x \in \mathbb{R} \,:\, x^2 + 3x + 3 = 0$		
$\forall\, x \in \mathbb{Z} \,:\, (-2)^{2x} > 0$		
$\forall\, x \in \mathbb{R}\ \exists\, y \in \mathbb{R} \,:\, 2x^2 = y^2 + 4$		
$\exists\, x \in \mathbb{R}\ \forall\, y \in \mathbb{R} \,:\, x^2 y - 3y = 0$		

Reminders:

- $\mathbb{R}$ denotes the set of all real numbers.
- $\mathbb{Z}$ denotes the set of all integers.
- $\forall$ denotes the universal quantifier ("for all") and $\exists$ denotes the existential quantifier ("there exists").

Problem 3: Prove by induction that the identity $\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$ holds for all integers $n \geq 0$. Show your work.