CS/EE 217 GPU Architecture and Parallel Programming

Lecture 12
Parallel Computation Patterns – Parallel Prefix Sum (Scan) Part-2

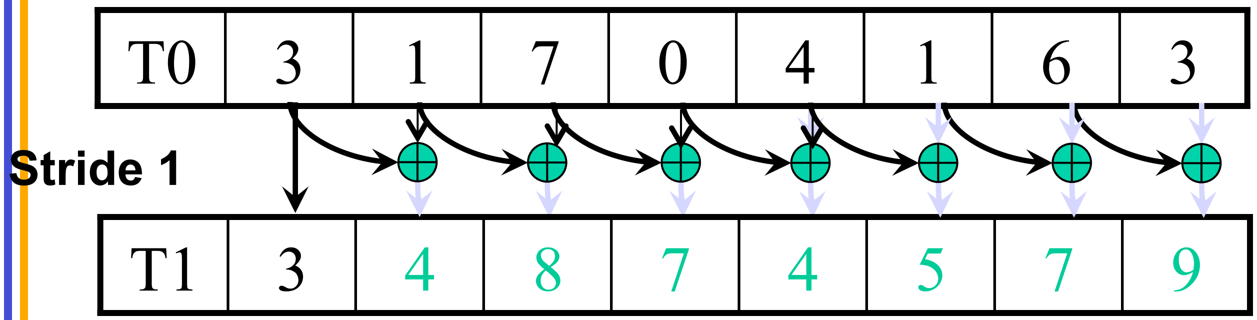
Recall: a Slightly Better Parallel Inclusive Scan Algorithm

T0	3	1	7	0	4	1	6	3

1. Read input from device memory to shared memory

Each thread reads one value from the input array in device memory into shared memory array T0. Thread 0 writes 0 into shared memory array.

A Slightly Better Parallel Scan Algorithm



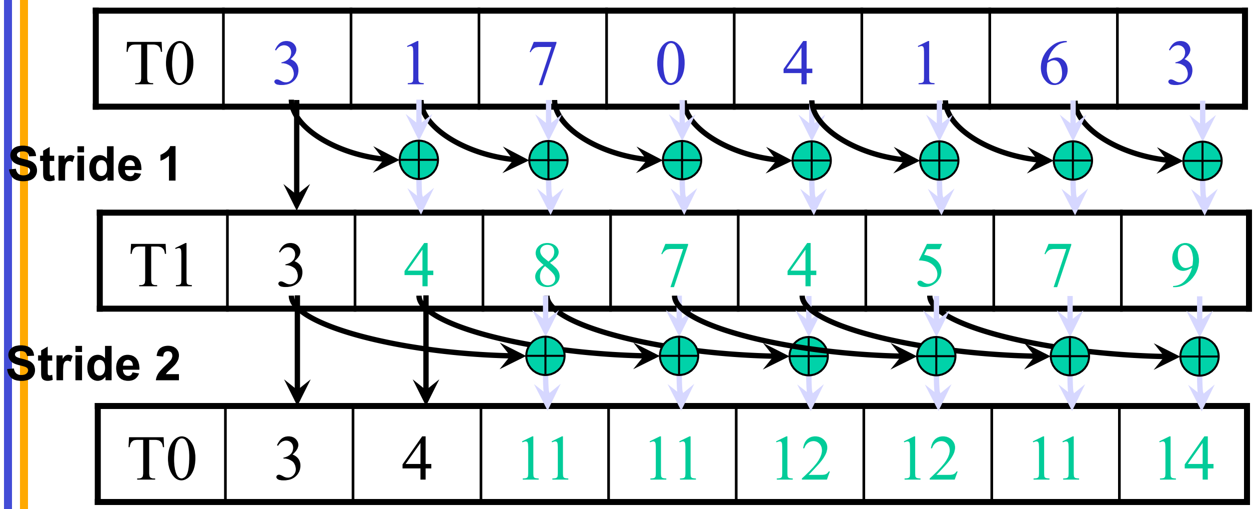
1. (previous slide)

2. Iterate log(n) times: Threads *stride* to *n:* Add pairs of elements *stride* elements apart. Double *stride* at each iteration. (note must double buffer shared mem arrays)

Iteration #1
Stride = 1

- Active threads: *stride* to *n*-1 (*n-stride* threads)
- Thread *j* adds elements *j* and *j-stride* from T0 and writes result into shared memory buffer T1 (ping-pong)

A Slightly Better Parallel Scan Algorithm

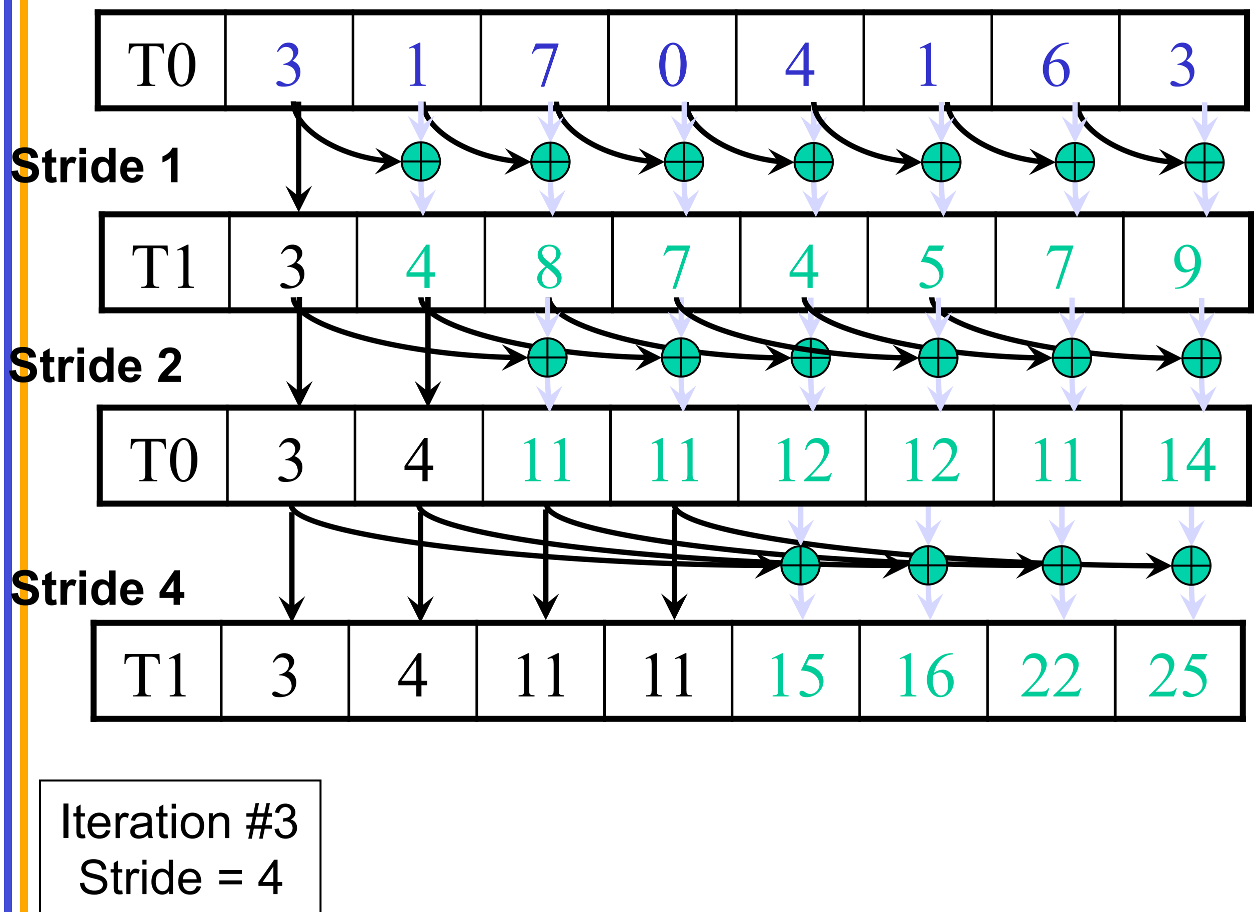


1. Read input from device memory to shared memory.

2. Iterate log(n) times: Threads *stride* to *n:* Add pairs of elements *stride* elements apart. Double *stride* at each iteration. (note must double buffer shared mem arrays)

Iteration #2
Stride = 2

A Slightly Better Parallel Scan Algorithm



1. Read input from device memory to shared memory. Set first element to zero and shift others right by one.

2. Iterate log(n) times: Threads *stride* to *n:* Add pairs of elements *stride* elements apart. Double *stride* at each iteration. (note must double buffer shared memory arrays)

3. Write output from shared memory to device memory

Work Efficiency Considerations

- The first-attempt Scan executes log(n) parallel iterations
  - The steps do (n-1), (n-2), (n-4),..(n- n/2) adds each
  - Total adds: n * log(n) - (n-1) → O(n*log(n)) work
- This scan algorithm is not very work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) hurts: 20x for 10^6 elements!
- A parallel algorithm can be slow when execution resources are saturated due to low work efficiency

Improving Efficiency

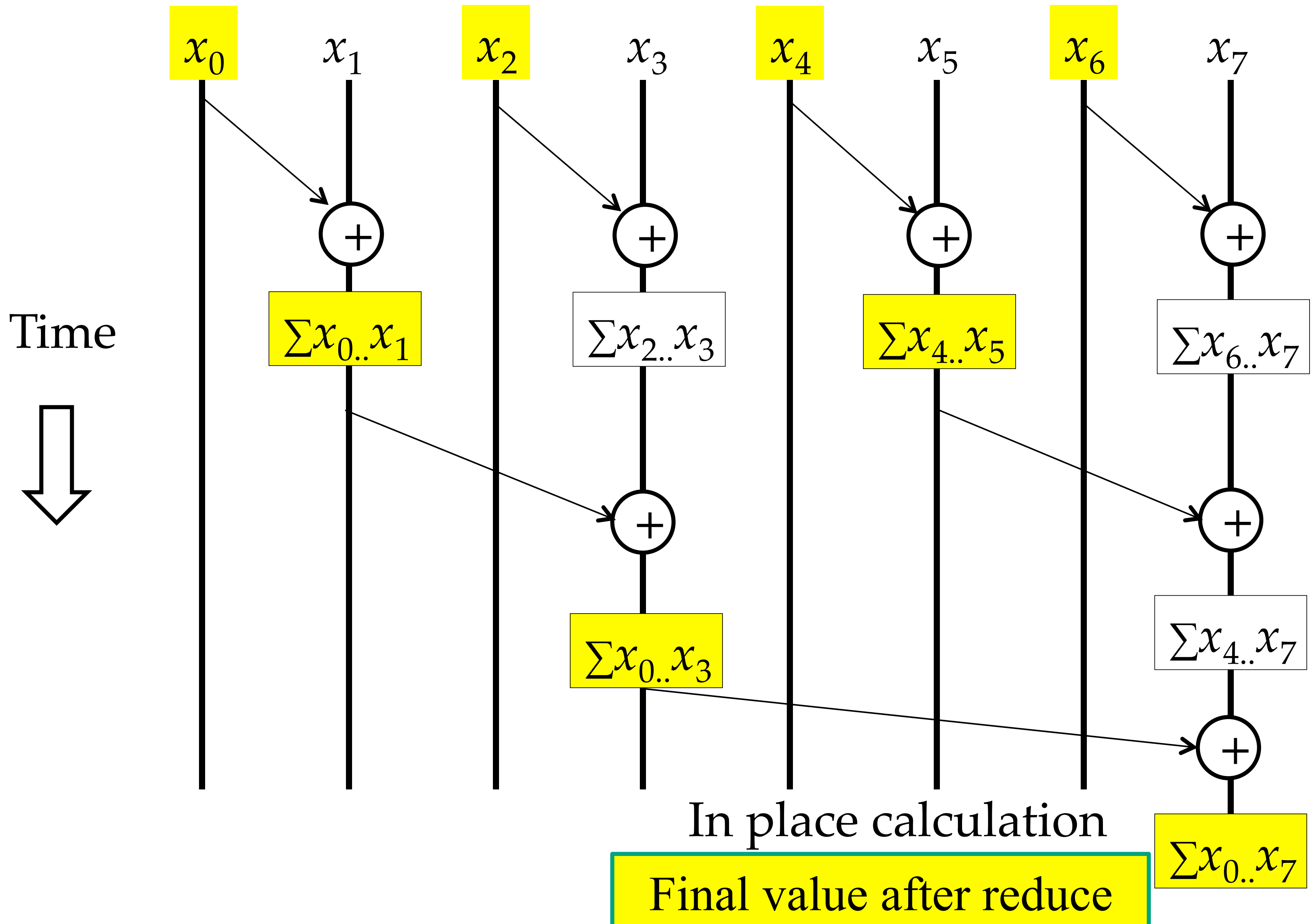
- A common parallel algorithm pattern:

  Balanced Trees

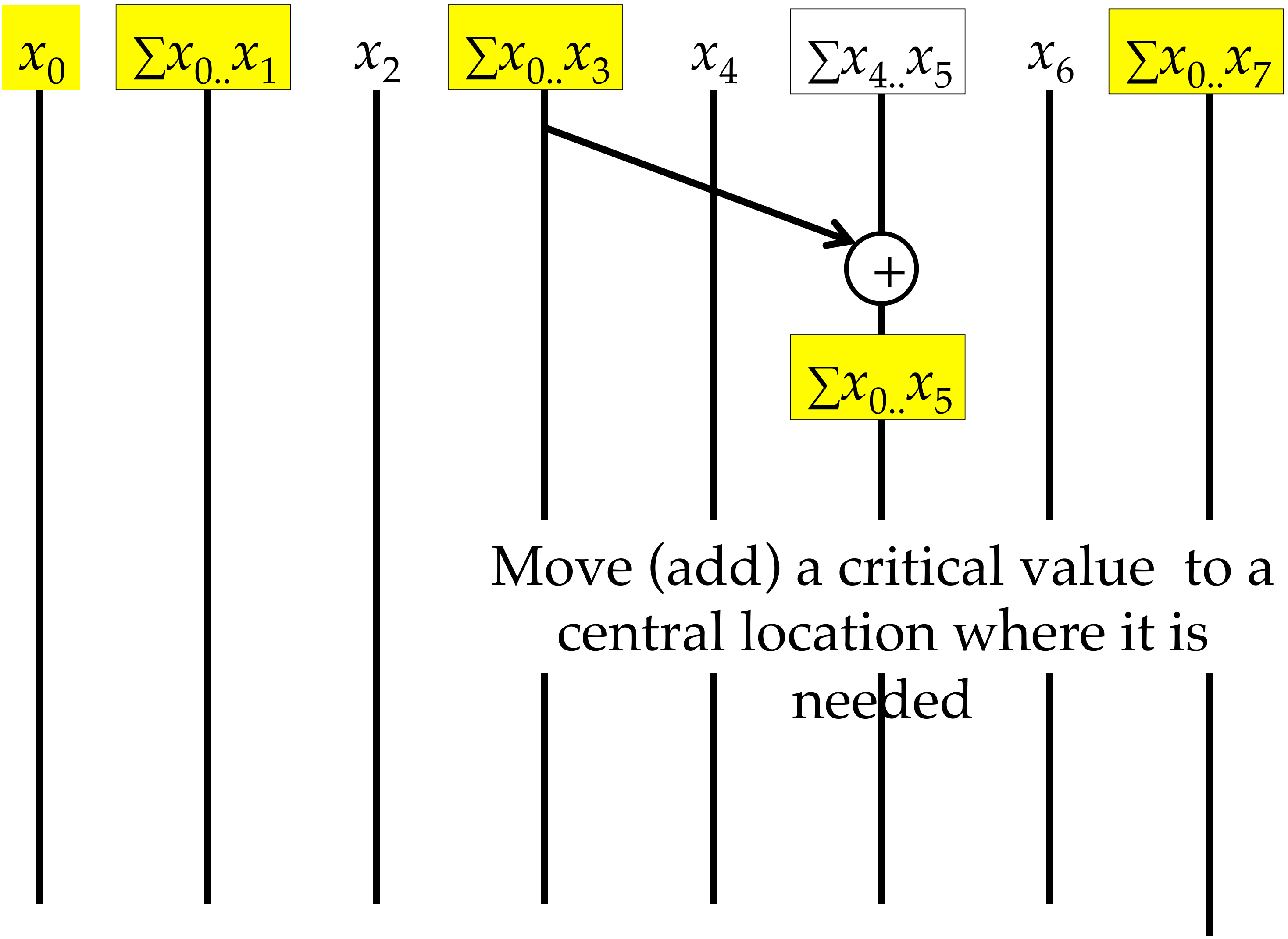
  - Build a balanced binary tree on the input data and sweep it to and from the root
  - Tree is not an actual data structure, but a concept to determine what each thread does at each step

- For scan:
  - Traverse down from leaves to root building partial sums at internal nodes in the tree
    - Root holds sum of all leaves
  - Traverse back up the tree building the scan from the partial sums

Parallel Scan - Reduction Step

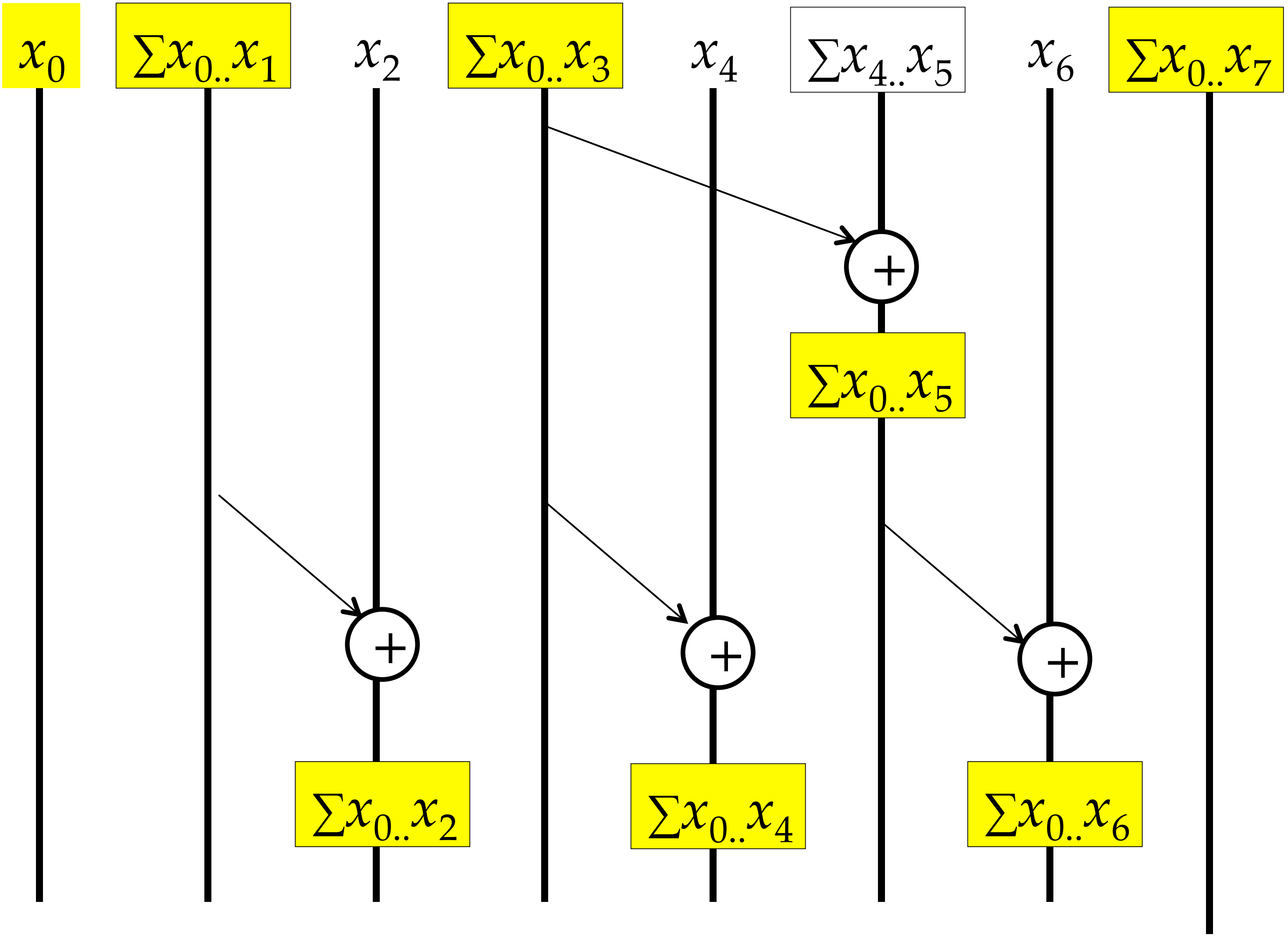


Inclusive Post Scan Step

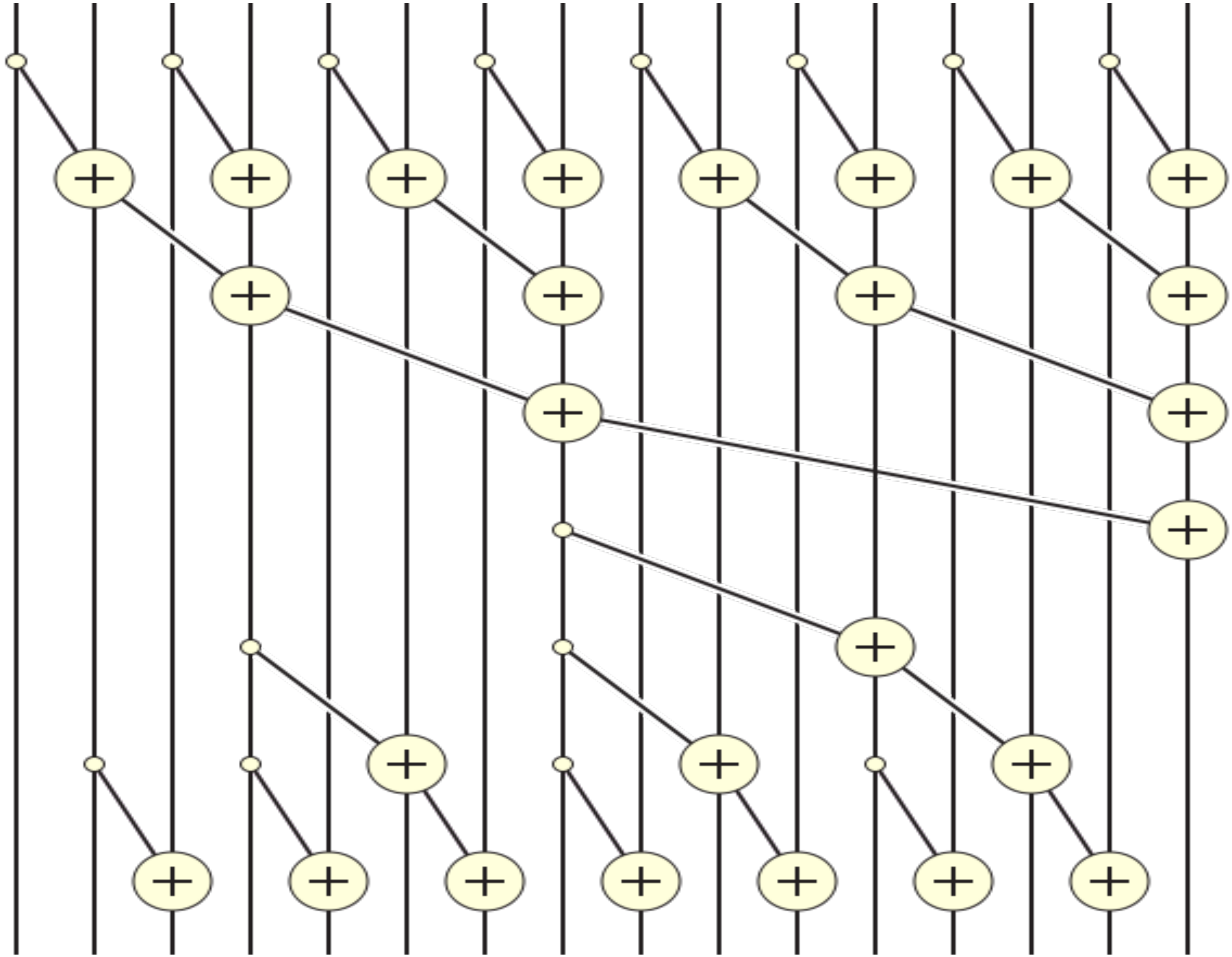


Move (add) a critical value to a central location where it is needed

Inclusive Post Scan Step



Putting it Together



Reduction Step Kernel Code

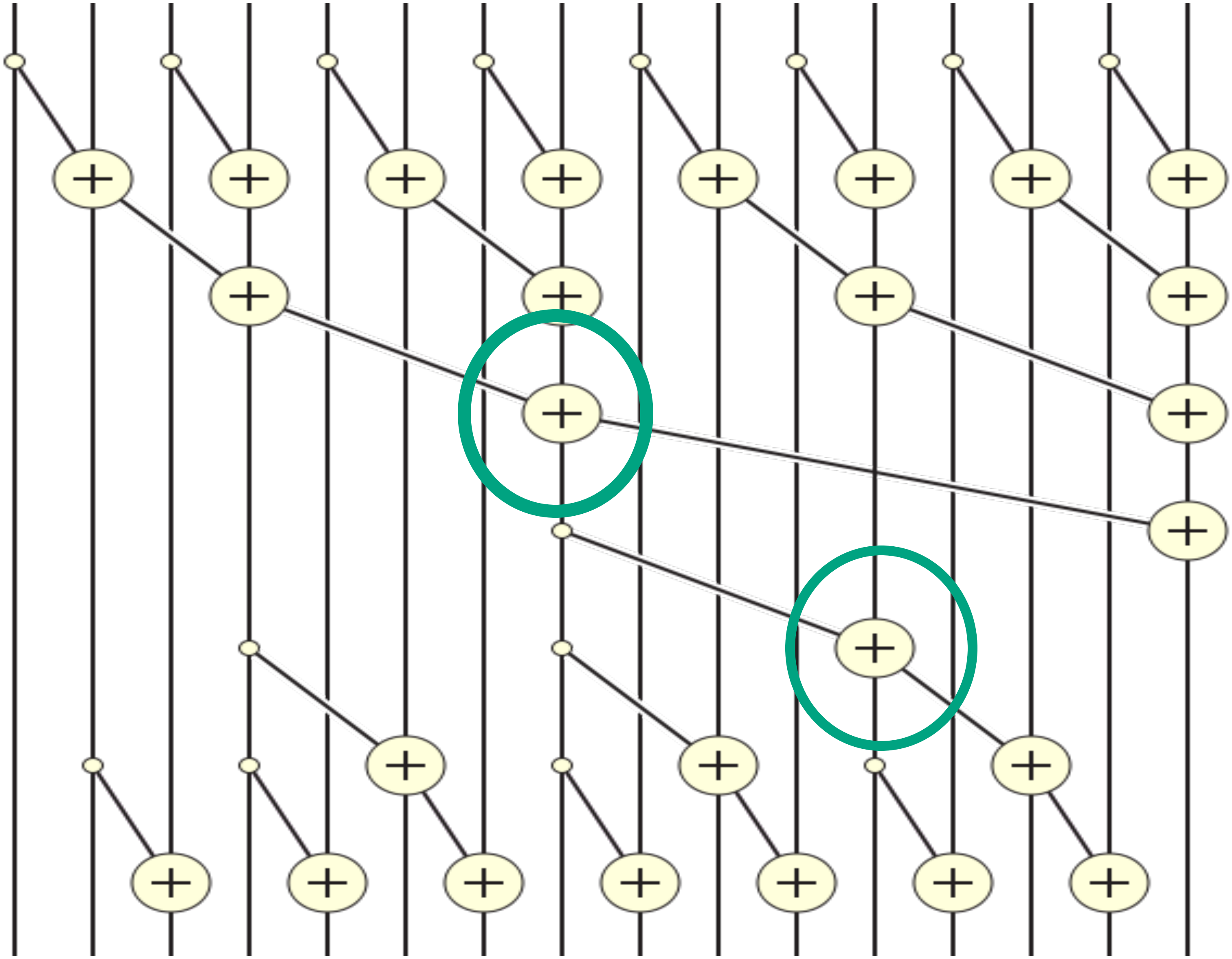
```
// XY[2*BLOCK_SIZE] is in shared memory

for(int stride=1; stride <= BLOCK_SIZE; stride=stride*2)
  {
     int index = (threadIdx.x+1)*stride*2 - 1;
     if(index < 2*BLOCK_SIZE)
        XY[index] += XY[index-stride];
     stride = stride*2;

     __syncthreads();
  }
```

threadIdx.x+1 = 1, 2, 3, 4….
stride = 1, index =

Putting it together



Post Scan Step

```
for(int stride=BLOCK_SIZE/2; stride > 0; stride /= 2)
    {
      __synchthreads();
       int index = (threadIdx.x+1)*stride*2 - 1;
       if(index + stride < 2*BLOCK_SIZE)
       {
          XY[index+stride] += XY[index];
       }
       stride = stride / 2;
}
 __syncthreads();
if(I < InputSize) Y[i] = XY[threadIdx.x];
```

Work Analysis

- Work efficient kernel executes log(n) iterations in reduction step
  - Identical to reduction; O(n) operations.
- log(n)-1 iterations in post reduction reverse step
  - 2-1, 4-1, 8-1, … n/2 -1 operations in each
  - Total? (n-1) – log(n) or O(n) work
- Both perform no more than 2*(n-1) adds
- Is this ok? What needs to happen for the parallel implementation to be better than sequential?

Some Tradeoffs

- Work efficient kernel is normally superior
  - Better energy efficiency (why?)
  - Less execution resource requirements

- However, the work inefficient kernel could be better under some special circumstances
  - What needs to happen for that?
  - Small lists where there are sufficient execution resources

(Exclusive) Prefix-Sum (Scan) Definition

Definition: *The* all-prefix-sums *operation takes a binary associative operator* $\oplus$*, and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}]$$

and returns the array

$$[0, x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})].$$

Example: If $\oplus$ is addition, then the all-prefix-sums operation on the array [3 1 7 0 4 1 6 3],
would return [0 3 4 11 11 15 16 22].

Why Exclusive Scan

- To find the beginning address of allocated buffers

- Inclusive and Exclusive scans can be easily derived from each other; it is a matter of convenience

	[3 1 7 0 4 1 6 3]
Exclusive	[0 3 4 11 11 15 16 22]
Inclusive	[3 4 11 11 15 16 22 25]

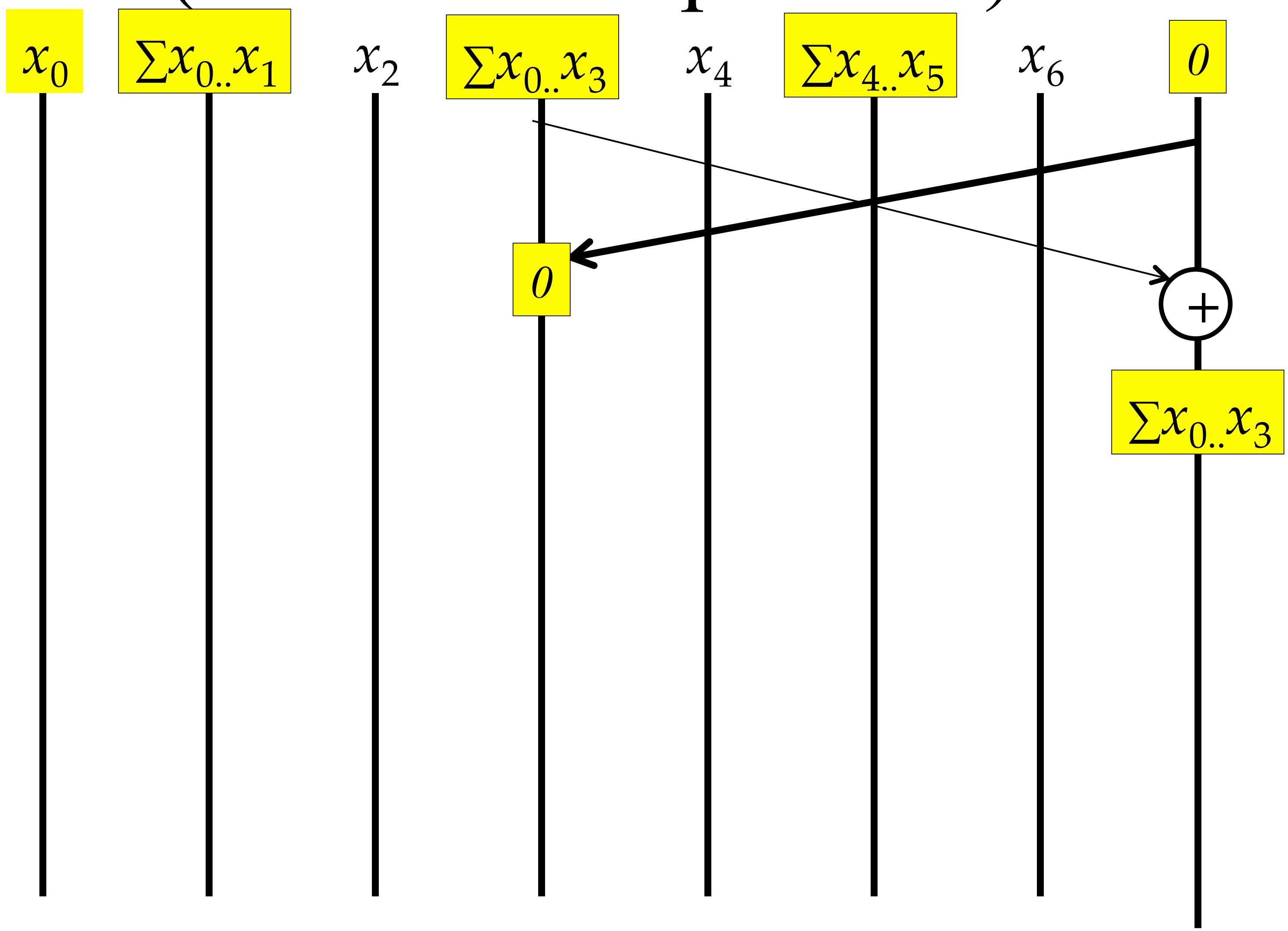
Simple exclusive scan kernel

- Adapt work inefficient scan kernel
- Block 0:
  - Thread 0 loads 0 in XY[0]
  - Other threads load X[threadIdx.x-1] into XY[threadIdx.x]
- All other blocks:
  - Load X[blockIdx.x*blockDim.x+threadIdx.x-1] into XY[threadIdx.x]
- Similar adaptation for work efficient kernel, except each thread loads two values – only one zero should be loaded

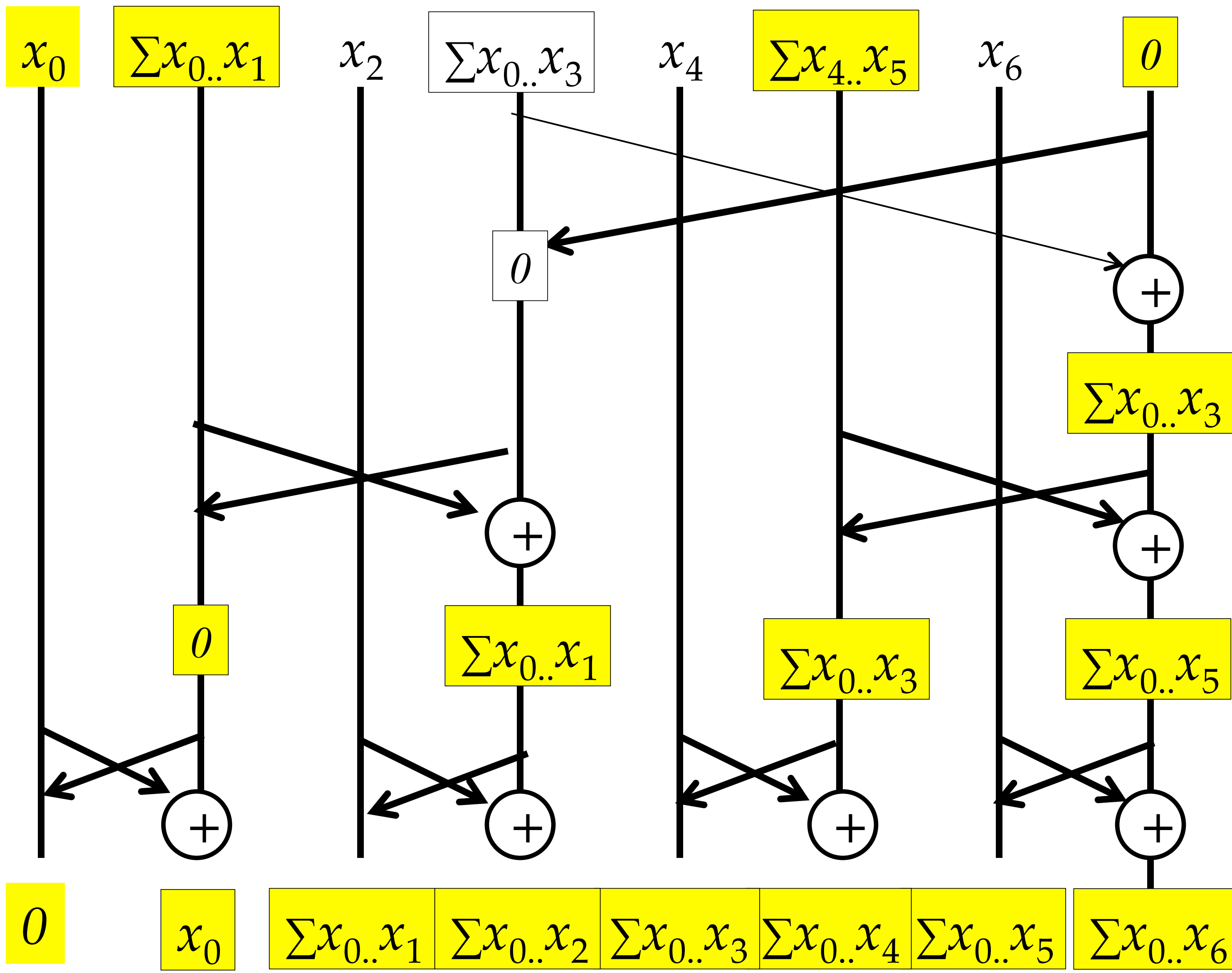
Alternative (read Harris Article)

- Uses add-move operation pairs
- Similar in complexity to the work efficient algorithm
- We’ll quickly overview

An Exclusive Post Scan Step (Add-move Operation)



Exclusive Post Scan Step

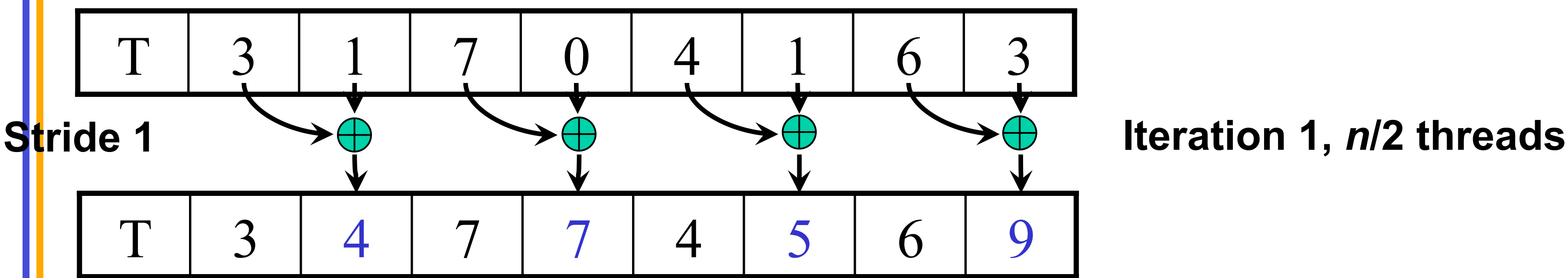


Exclusive Scan Example – Reduction Step

T	3	1	7	0	4	1	6	3

Assume array is already in shared memory

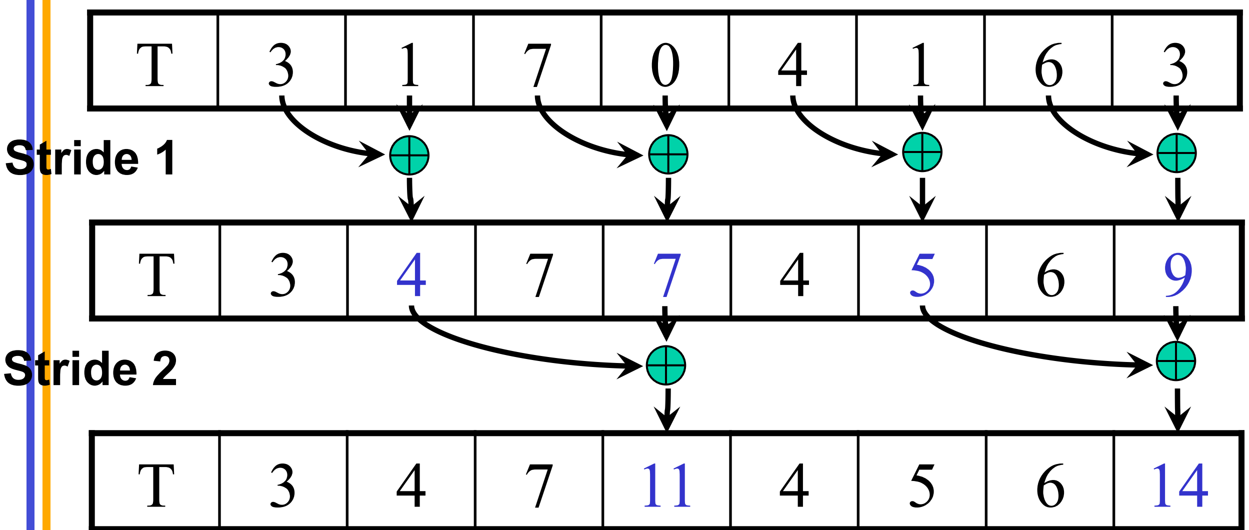
Reduction Step (cont.)



Each $\oplus$ corresponds to a single thread.

Iterate log(n) times. Each thread adds value *stride* elements away to its own value

Reduction Step (cont.)

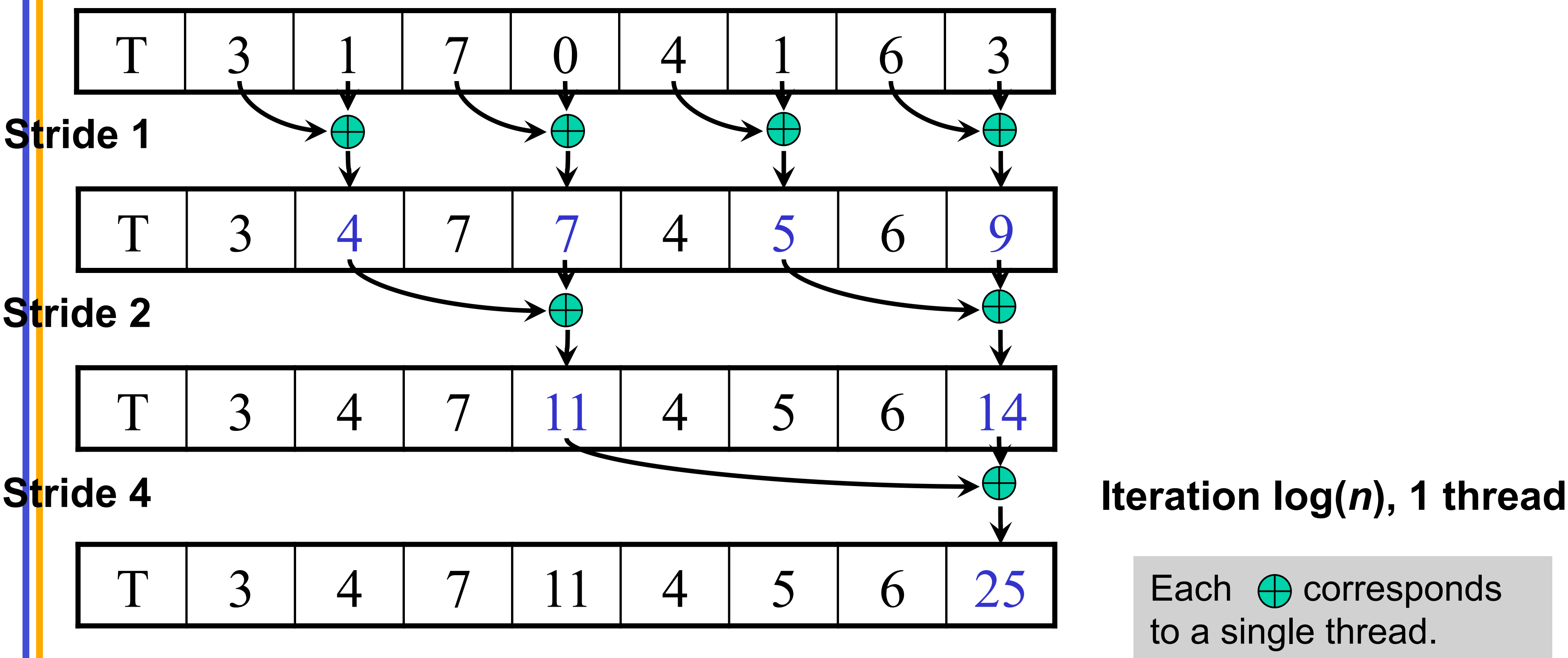


Iteration 2, *n*/4 threads

Each ⊕ corresponds to a single thread.

Iterate log(n) times. Each thread adds value *stride* elements away to its own value

Reduction Step (cont.)



Iterate log(n) times. Each thread adds value *stride* elements away to its own value.

Note that this algorithm operates in-place: no need for double buffering

Zero the Last Element

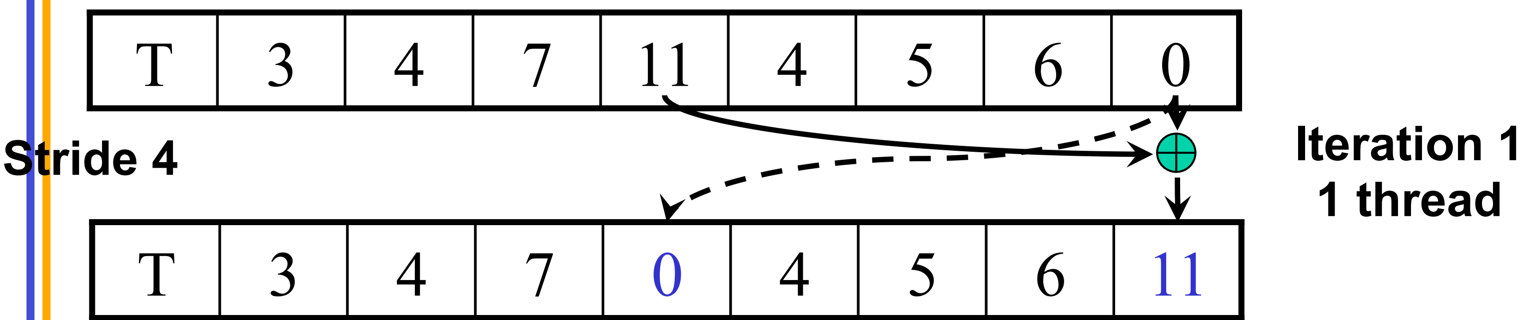
T	3	4	7	11	4	5	6	0

We now have an array of partial sums. Since this is an exclusive scan, set the last element to zero. It will propagate back to the first element.

Post Scan Step from Partial Sums

T	3	4	7	11	4	5	6	0

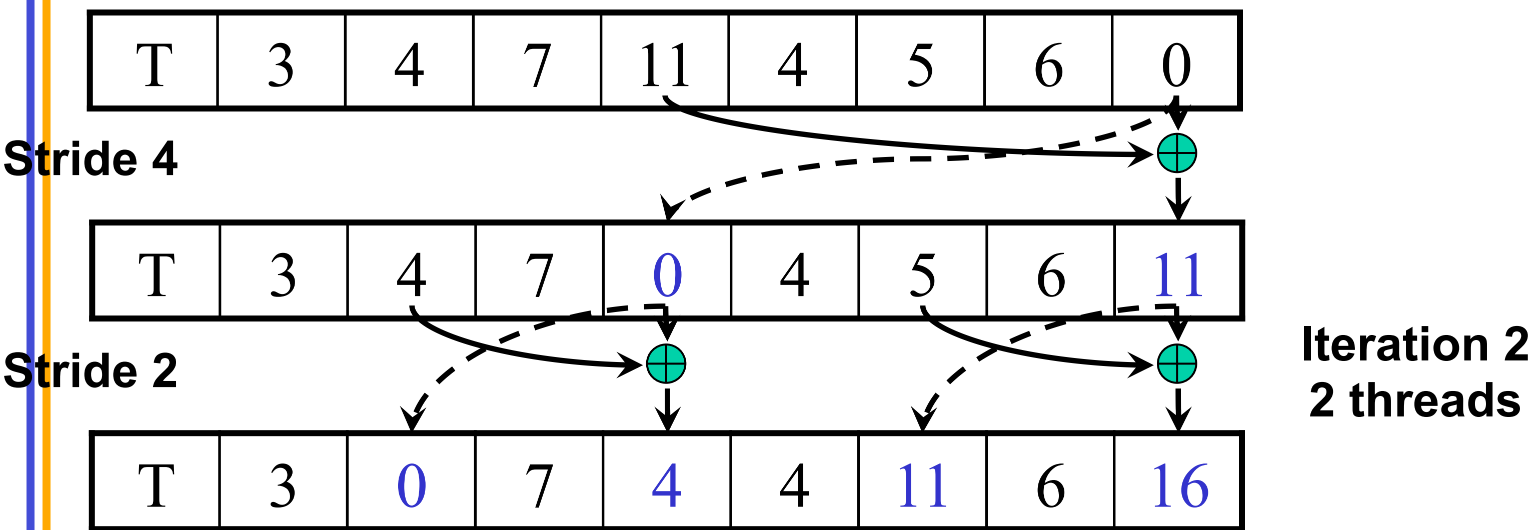
Post Scan Step from Partial Sums (cont.)



Each $\oplus$ corresponds to a single thread.

Iterate log(n) times. Each thread adds value *stride* elements away to its own value, and sets the value *stride* elements away to its own *previous* value.

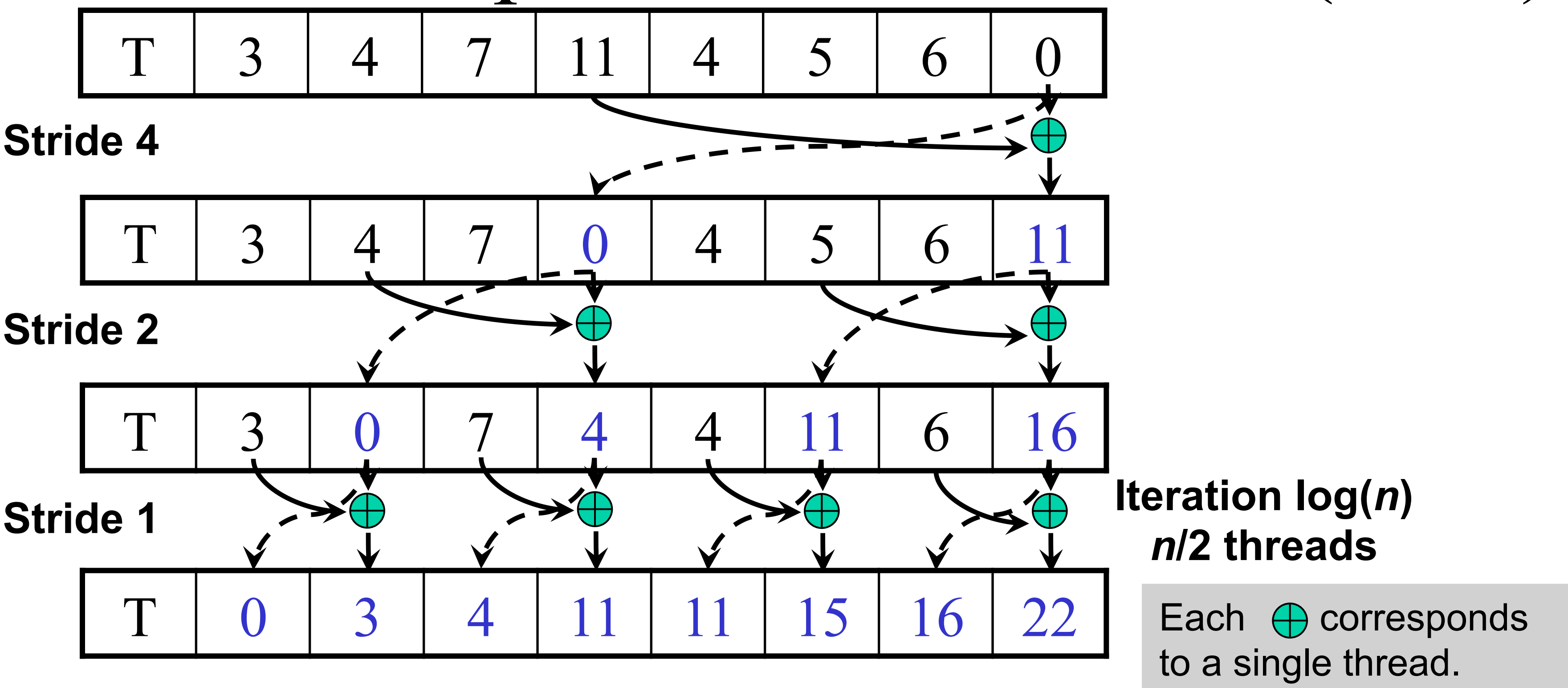
Post Scan From Partial Sums (cont.)



Each ⊕ corresponds to a single thread.

Iterate log(n) times. Each thread adds value *stride* elements away to its own value, and sets the value *stride* elements away to its own *previous* value.

Post Scan Step From Partial Sums (cont.)



Done! We now have a completed scan that we can write out to device memory.

Total steps: 2 * log(*n*).
Total work: 2 * (*n*-1) adds = *O*(*n*) **Work Efficient!**

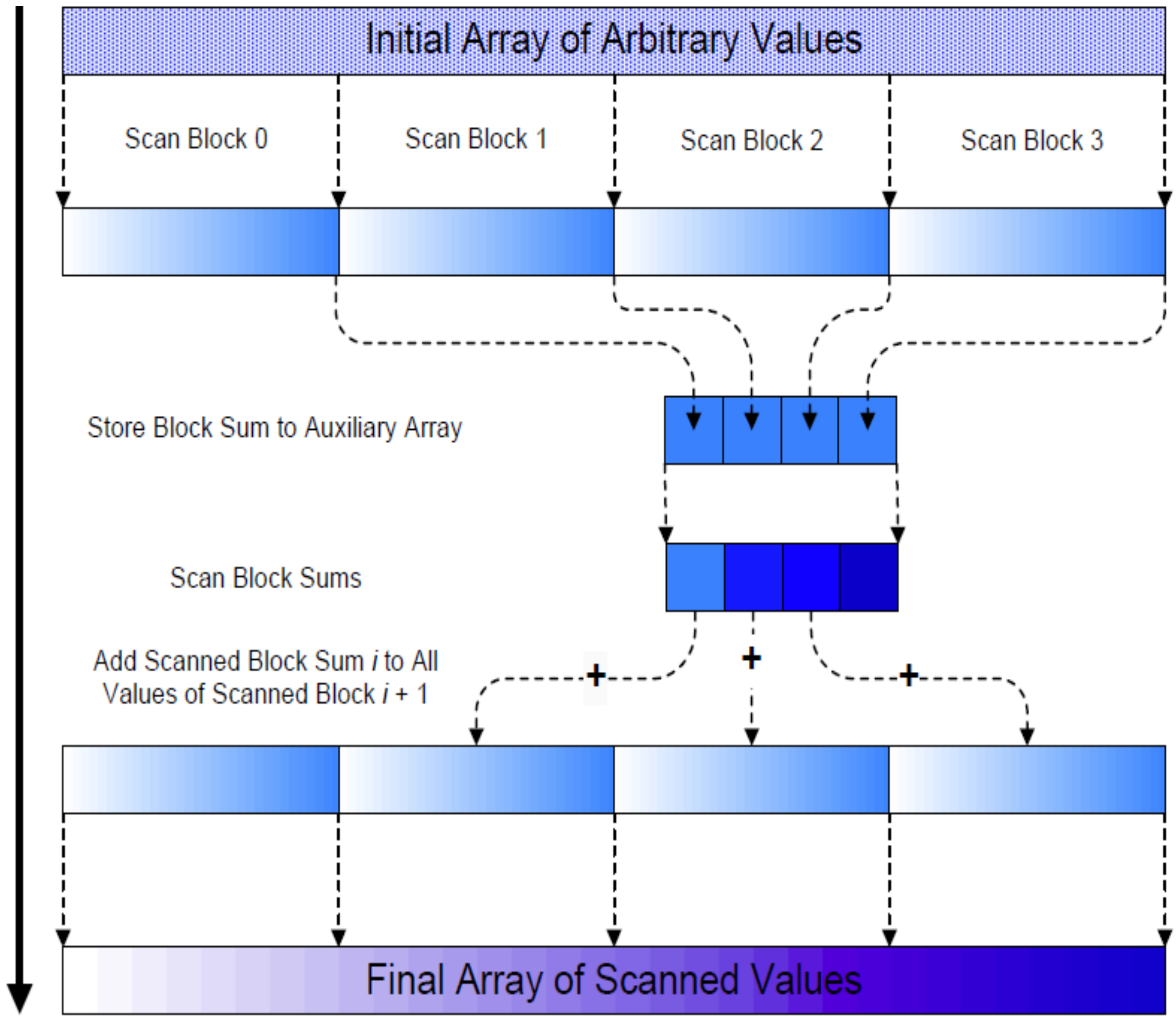
Work Analysis

- The parallel Inclusive Scan executes 2* log(n) parallel iterations
  - log(n) in reduction and log(n) in post scan
  - The iterations do n/2, n/4,..1, 1, …., n/4. n/2 adds
  - Total adds: 2* (n-1) → O(n) work
- The total number of adds is no more than twice of that done in the efficient sequential algorithm
  - The benefit of parallelism can easily overcome the 2X work when there is sufficient hardware

Working on Arbitrary Length Input

- Build on the scan kernel that handles up to 2*blockDim.x elements
- Have each section of 2*blockDim elements assigned to a block
- Have each block write the sum of its section into a Sum array indexed by blockIdx.x
- Run parallel scan on the Sum array
  - May need to break down Sum into multiple sections if it is too big for a block
- Add the scanned Sum array values to the elements of corresponding sections

Overall Flow of Complete Scan



ANY MORE QUESTIONS? READ CHAPTER 9