Name: **SID:** **LAB Section:**

Lab 10 - Part 1: Bézier curves

In this lab, we will render an approximation of a parametric curve known as the Bézier.

Consider the parametric equation of a segment between two **control points** P0 and P1:

(1) B(t) = (1 - t) * P0 + t * P1

For n control points, we can recursively apply Eq. 1 to consecutive control points until we are left with only P(t). For three control points:

(2) B(t) = **(1 - t)** * [**(1 - t)** * P0 + **t** * P1]
+ **t** * [**(1 - t)** * P1 + **t** * P2]

1. Given n control points, what is the degree of the polynomial equation for the Bezier curve?

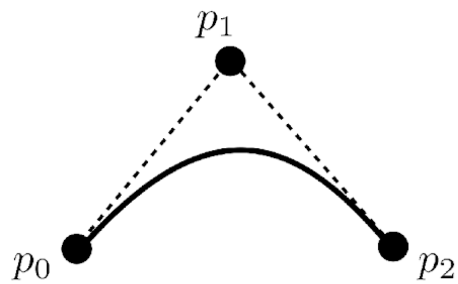
In general, B(t) for n points is given by:

$$B(t) = \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} P_i$$

2. Since we may need the **factorial**, **combination** and **binomial** terms of B(t) in this lab, complete the code to for these functions below.

```
float factorial(int n) {




}
float combination(int n, int k) {


}

float binomial(int n, int k, float t) {


}
```

3. Let's practice a problem on CS130 final exam from the last quarter.



19. *Quadratic Bezier curve.* This problem will guide you through deriving the quadratic Bezier blending functions.

Given three control points p_0, p_1, and p_2, a quadratic Bezier curve

$$f(u) = a_0 + a_1 u + a_2 u^2 \tag{1}$$

can be determined from the following conditions:

condition 1 $f(0) = p_0$

condition 2 $f(1) = p_1$

condition 3 $f'(0) = 2(p_1 - p_0)$

(a) Fill in the right hand side of the equation below by differentiating equation (1).

$$f'(u) = \tag{2}$$

(b) Use conditions 1-3 and equations (1) and (2) to fill in the following linear system:

$$\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \end{pmatrix}$$

(c) Given that

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a & -b & 1 \end{pmatrix}$$

fill in the following linear system:

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \end{pmatrix}$$

(d) Use the above work to write down the quadratic Bezier blending functions $b_o(u)$, $b_1(u)$, $b_2(u)$, such that

$$f(u) = b_o(u)p_0 + b_1(u)p_1 + b_2(u)p_2.$$

(hint: recall that $f(u) = \mathbf{u}^T\mathbf{a}$, where $\mathbf{u} = (1, u, u^2)^T$ and $\mathbf{a} = (a_0, a_1, a_2)^T$.)

Extra space:

Lab 10 - Part 2: Coding

Download the skeleton code from iLearn and modify **main.cpp** as follows:

- ❑ **Define a global vector** to store the control points.
- ❑ **Push back the mouse click coordinates** into the vector in the function GL_mouse.
- ❑ Write the code for the **factorial, combination and binomial.**
- ❑ **Draw line segments between points along the Bezier curve** in GL_render().
    - ❑ You can use **GL_LINE_STRIP** to draw line segments between consecutive points.
    - ❑ Iterate **t** between 0 and 1 in steps of 0.01.

Optional: Rather than using the general equation for the Bézier curve to write your program, can write a program where you recursively apply Eq. 1 to consecutive points until get B(t)?

We hope very much the labs have been fun and of some help to you. We sincerely hope you can use what you learned with us to succeed in the future. You can help us out by leaving a feedback on iEval so we can continue to improve the course (deadline is Friday, March 16). Cheers!