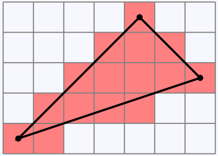
Line Rasterization

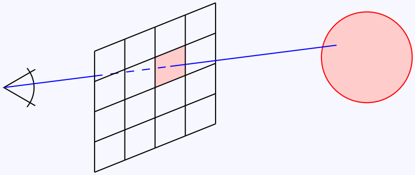
University of California Riverside

Raster Image

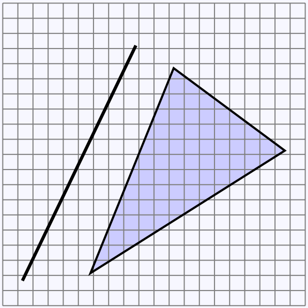
- Object oriented
  - for each object…



- Image oriented
  - for each pixel…



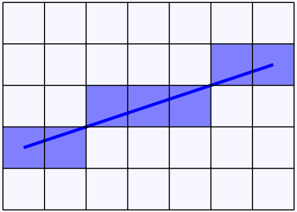
What is rasterization?



Rasterization is the process of determining which pixels are "covered" by the primitive

Rasterization

- In: 2D primitives (floating point)
- Out: covered pixels (integer)
- Must be fast (called **many times**)
- Visually pleasing
  - lines have constant width
  - lines have no gaps



DDA algorithm for lines

- DDA = “digital differential analyzer”

DDA algorithm for lines

- DDA = "digital differential analyzer"
- Plot line $y = mx + b$

DDA algorithm for lines

- DDA = “digital differential analyzer”
- Plot line $y = mx + b$
- For each x:

DDA algorithm for lines

- DDA = "digital differential analyzer"
- Plot line $y = mx + b$
- For each x:
  - $y = mx + b$

DDA algorithm for lines

- DDA = "digital differential analyzer"
- Plot line $y = mx + b$
- For each x:
  - $y = mx + b$
  - turn on pixel $(x, \text{round}(y))$

DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right

DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$

DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$

$$\begin{aligned} y_{i+1} &= mx_{i+1} + b \\ &= m(x_i + 1) + b \\ &= y_i + m \end{aligned}$$

DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$

$$\begin{aligned} y_{i+1} &= mx_{i+1} + b \\ &= m(x_i + 1) + b \\ &= y_i + m \end{aligned}$$

- Each time:

DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$

$$
\begin{aligned}
y_{i+1} &= m x_{i+1} + b \\
&= m(x_i + 1) + b \\
&= y_i + m
\end{aligned}
$$

- Each time:
  - Increment x

DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$

$$\begin{aligned} y_{i+1} &= m x_{i+1} + b \\ &= m(x_i + 1) + b \\ &= y_i + m \end{aligned}$$

- Each time:
  - Increment x
  - Add m to y

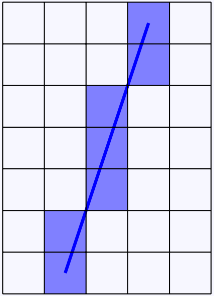
DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right
  - $x_0 = \text{start}$, $x_{i+1} = x_i + 1$, $x_n = \text{end}$

$$\begin{aligned} y_{i+1} &= mx_{i+1} + b \\ &= m(x_i + 1) + b \\ &= y_i + m \end{aligned}$$

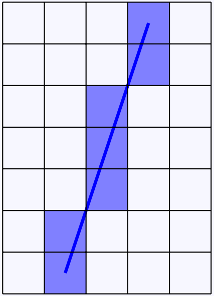
- Each time:
  - Increment x
  - Add m to y
  - turn on pixel $(x_i, \text{round}(y_i))$

DDA algorithm for lines



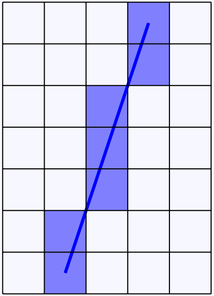
- What if $|m| > 1$?

DDA algorithm for lines



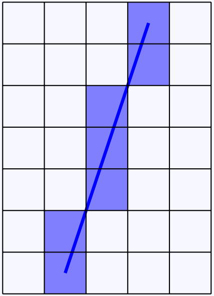
- What if $|m| > 1$?
- Increment y by m

DDA algorithm for lines



- What if $|m| > 1$?
- Increment y by m
- round(y) may skip an integer
  - gap in the line

DDA algorithm for lines



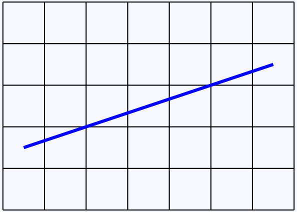
- What if $|m| > 1$?
- Increment y by m
- round(y) may skip an integer
  - gap in the line
- Swap the roles of x and y
  - Loop over y, compute and round x

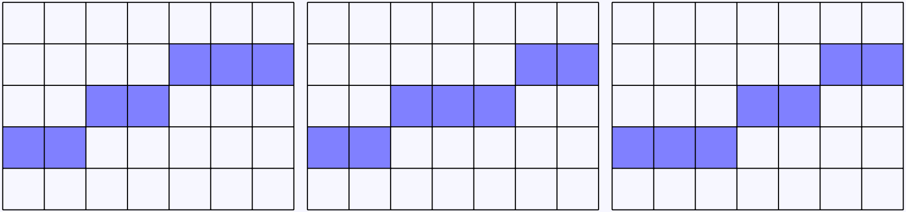
DDA algorithm for lines - limitations

- Must round for each pixel
  - very slow
- Only use ops: $+, -, \times$
  - Even better: $+, -$

Rasterization choices

- Thin, no gaps
- Still have choices

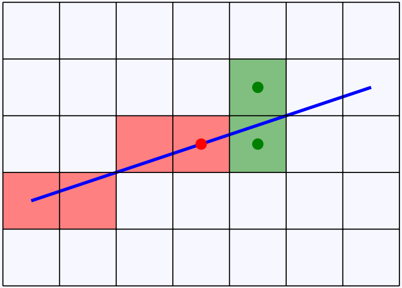




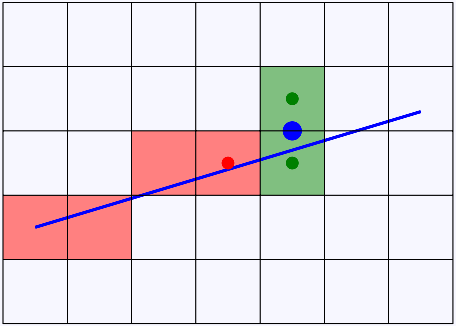
Midpoint algorithm

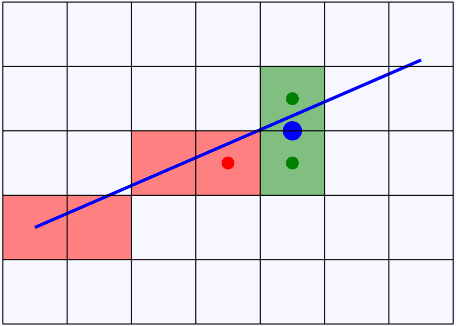
- Assume $0 \leq m \leq 1$
- Move from left to right
- Choose between $(x+1, y)$ and $(x+1, y+1)$

$y = y_0$
for $x = x_0, \ldots, x_1$ **do**
  draw(x, y)
  if $\langle$condition$\rangle$ **then**
    $y \leftarrow y + 1$

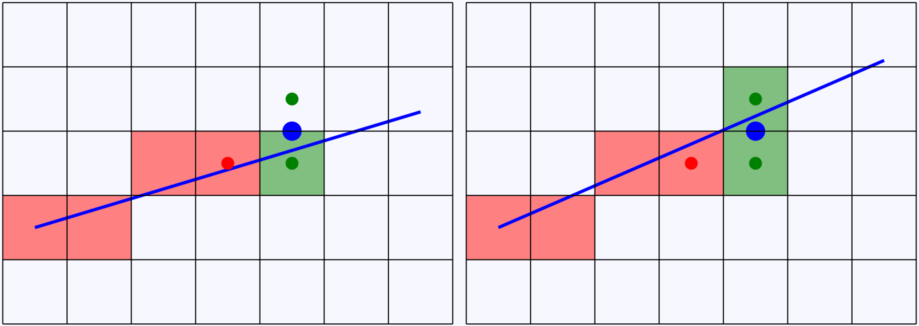


Check midpoint location

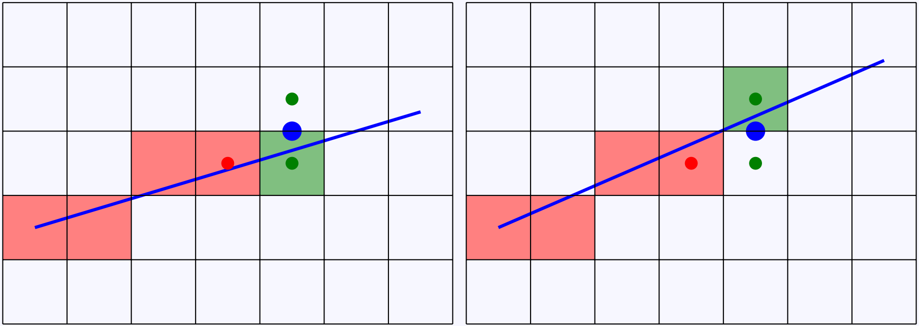




Check midpoint location



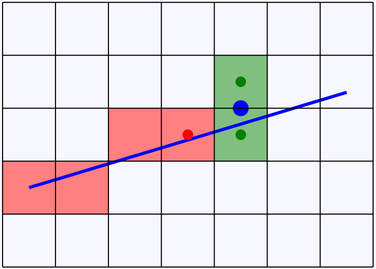
Check midpoint location



Criterion

Implicit line equation:

$$f(\mathbf{x}) = \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0$$



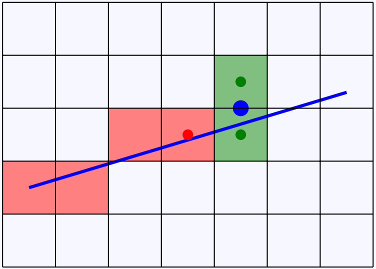
Criterion

Implicit line equation:

$$f(\mathbf{x}) = \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0$$

Evaluate f at midpoint:

$$f\left(x + 1, y + \frac{1}{2}\right) \overset{?}{<} 0$$



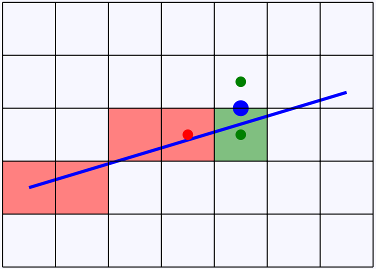
Criterion

Implicit line equation:

$$f(\mathbf{x}) = \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0$$

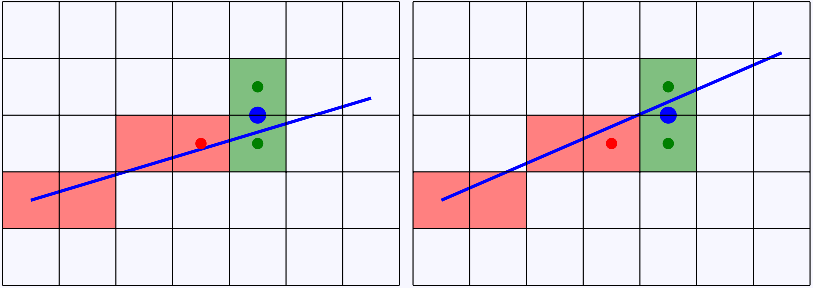
Evaluate f at midpoint:

$$f\left(x+1, y+\frac{1}{2}\right) < 0$$



Midpoint algorithm ($0 \le m \le 1$)

$y \leftarrow y_0$
for $x = x_0, \ldots, x_1$ **do**
  draw(x, y)
  if $f(x+1, y+\frac{1}{2}) < 0$ **then**
    $y \leftarrow y + 1$



Efficiency: incremental update

- Compute initial $f(x, y)$
- Compute next by updating previous
- Update with *one* addition

$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + (x_0 y_1 - x_1 y_0)$$

Efficiency: incremental update

- Compute initial $f(x, y)$
- Compute next by updating previous
- Update with *one* addition

$$
\begin{aligned}
f(x, y) &= (y_0 - y_1)x + (x_1 - x_0)y + (x_0 y_1 - x_1 y_0) \\
f(x+1, y) &= f(x, y) + (y_0 - y_1)
\end{aligned}
$$

Efficiency: incremental update

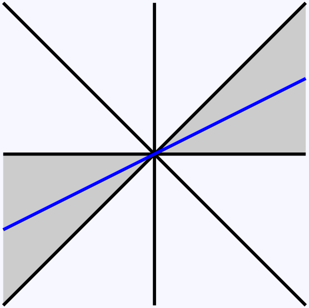
- Compute initial $f(x, y)$
- Compute next by updating previous
- Update with *one* addition

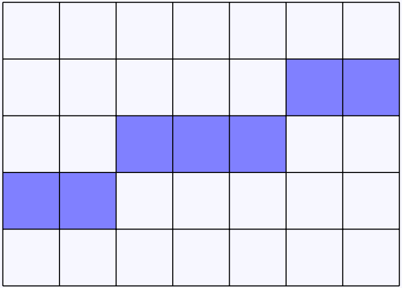
$$
\begin{aligned}
f(x, y) &= (y_0 - y_1)x + (x_1 - x_0)y + (x_0 y_1 - x_1 y_0) \\
f(x + 1, y) &= f(x, y) + (y_0 - y_1) \\
f(x + 1, y + 1) &= f(x, y) + (y_0 - y_1) + (x_1 - x_0)
\end{aligned}
$$

Efficiency: incremental update

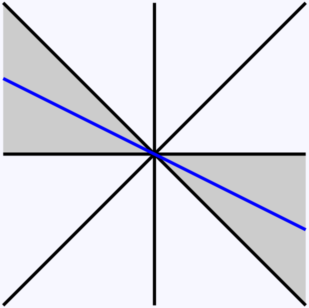
$y \leftarrow y_0$
$d \leftarrow f(x_0 + 1, y_0 + \frac{1}{2})$
for $x = x_0, \ldots, x_1$ **do**
    draw(x, y)
    if $d < 0$ **then**
        $y \leftarrow y + 1$
        $d \leftarrow d + (y_0 - y_1) + (x_1 - x_0)$
    else
        $d \leftarrow d + (y_0 - y_1)$

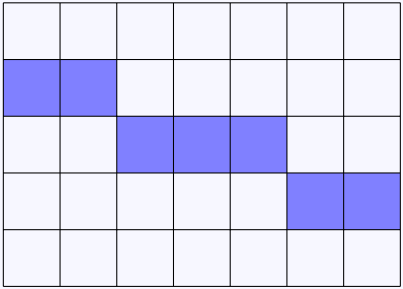
Other cases: $0 \leq m \leq 1$





Other cases: $-1 \le m \le 0$





Other cases: $|m| > 1$