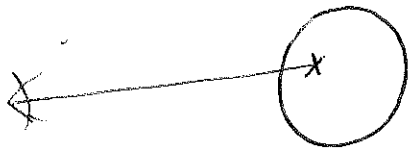
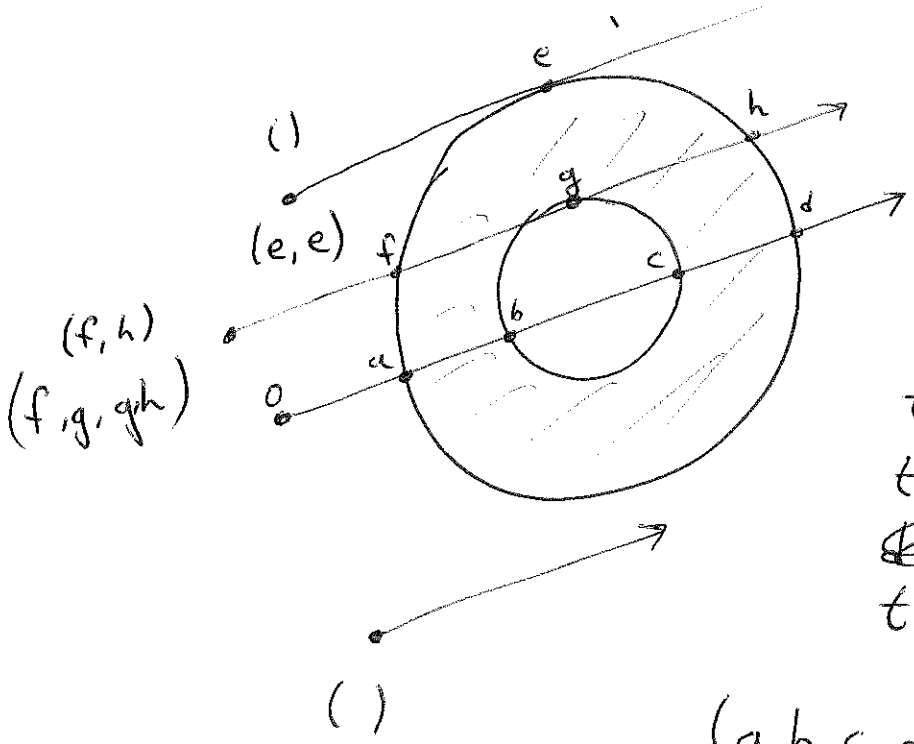




E = enter

L = leave

t=a E

t=b L

t=c E

t=d L

( )

(e)

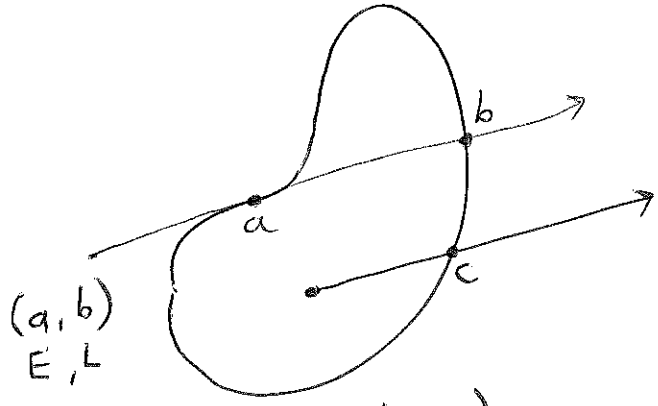
(e, e)

(f, h)

(f, g, gh)

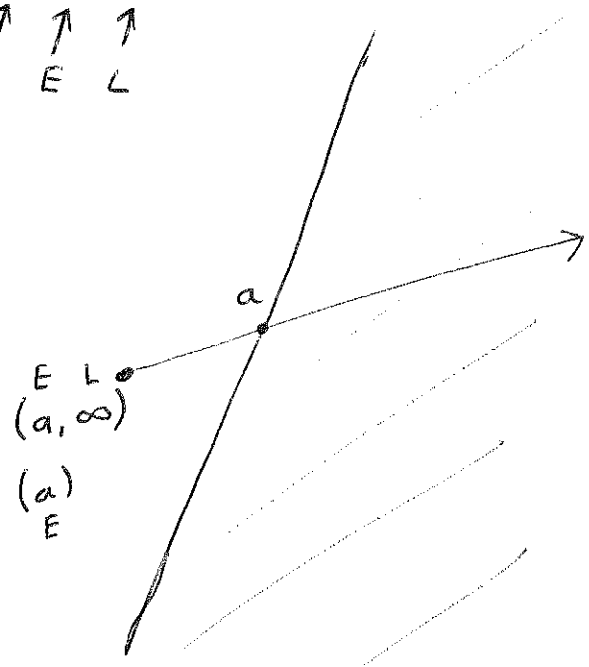
(a, b, c, d)

E L E L



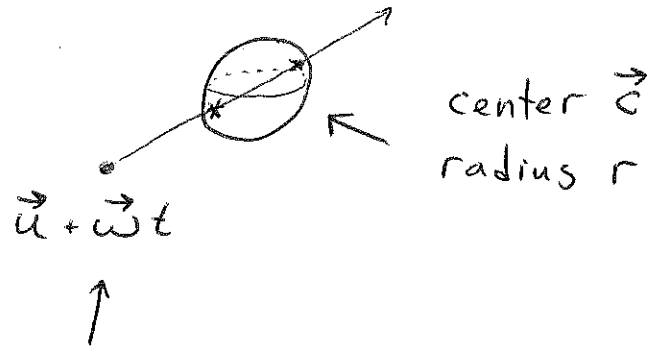
(a, b)
E, L

(0, c)



E L
(a, ∞)

(a)
E



$\|\vec{\omega}\| = 1$

on sphere: $\|\vec{x} - \vec{c}\| = r$

$\|\vec{x} - \vec{c}\|^2 = r^2$

$(\vec{x} - \vec{c}) \cdot (\vec{x} - \vec{c}) = r^2$ $\qquad \vec{x} = \vec{u} + \vec{\omega} t$

$(\omega t + \underbrace{(u - c)}_{v}) \cdot (\omega t + (u - c)) = r^2$ $\qquad v = u - c$

$(\omega t + v) \cdot (\omega t + v) = r^2$

$(\omega \cdot \omega) t^2 + 2 (\omega \cdot v) t + (v \cdot v - r^2) = 0$

$$t = \frac{(\omega \cdot v) \pm \sqrt{(\omega \cdot v)^2 - (\omega \cdot \omega)(v \cdot v - r^2)}}{\omega \cdot \omega}$$

← discriminant $= D$

$\omega \cdot \omega = 1$

if $D < 0$ no intersection $\Rightarrow$ ( )

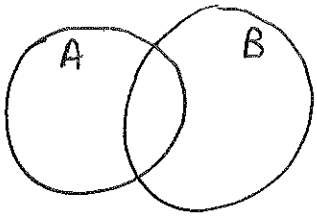
if $D = 0$ duplicate $\Rightarrow$ $(t, t)^{(2)}$

if $D > 0$ (t_0, t_1) assume $t_0 \leq t_1$

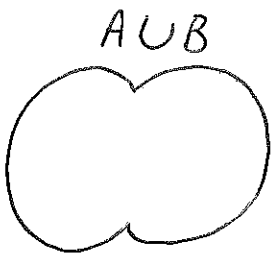
if $t_0 \leq t_1 < 0 \Rightarrow$ ( )

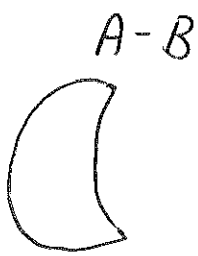
$t_0 < 0 \leq t_1 \Rightarrow (0, t_1)$

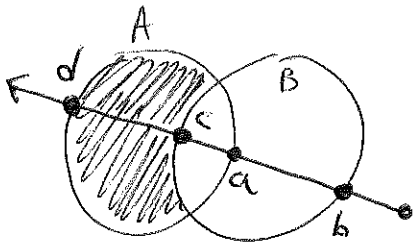
$0 \leq t_0 \leq t_1 \Rightarrow (t_0, t_1)$

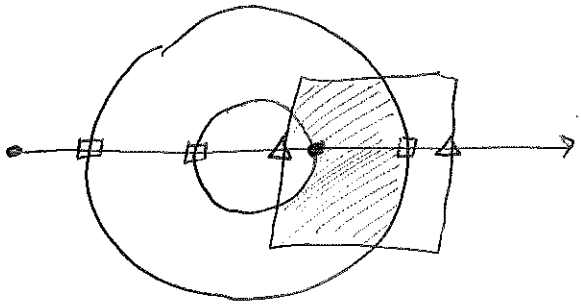

A
B

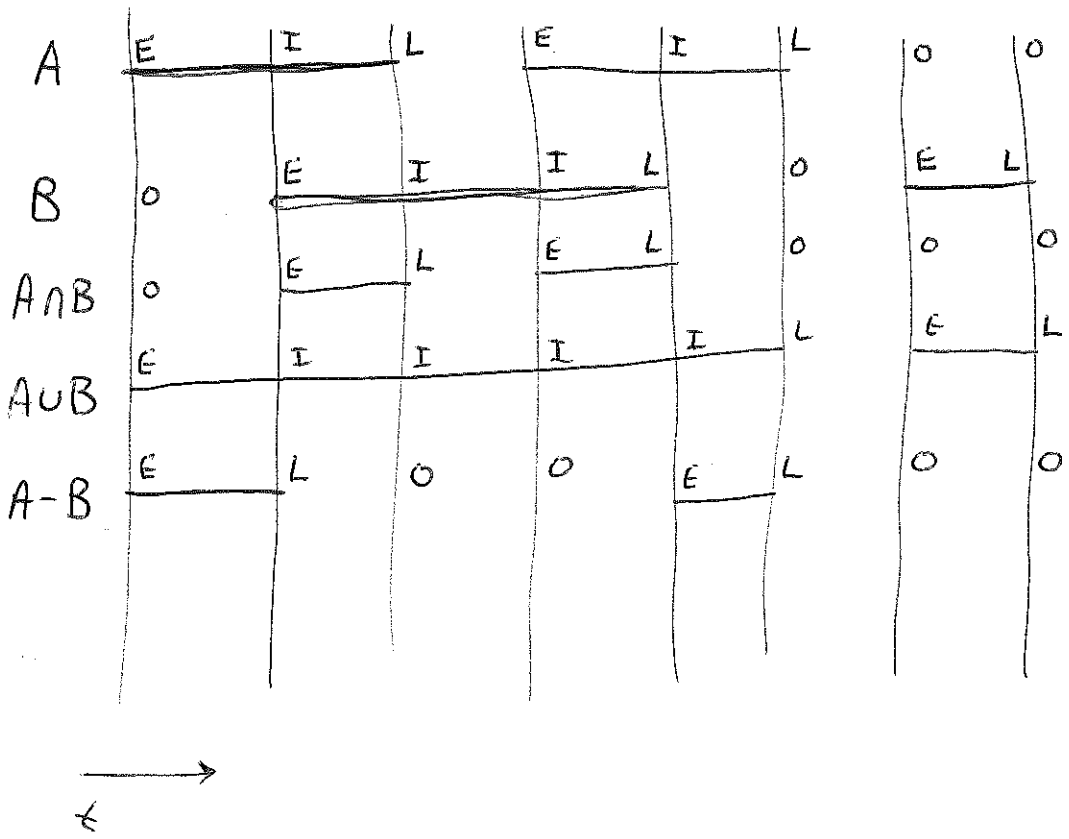

A ∩ B

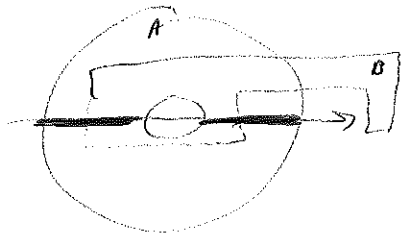

A ∪ B


A - B


A
B
d
c
a
b







E = enter
L = leave
I = inside
O = outside

t →

A	B	A∩B	A∪B	A-B
I	E	E	I	L
I	L	L	I	E
O	E	O	E	O
O	L	O	L	O
E	I	E	I	O
E	O	O	E	E
L	I	L	I	O
L	O	O	L	L