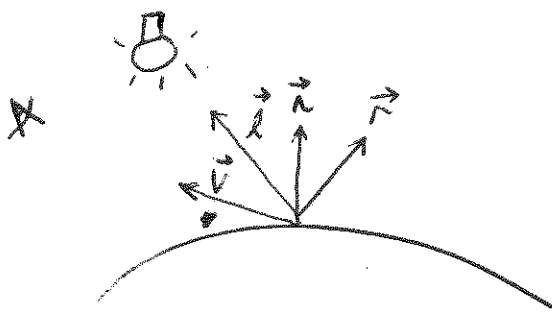
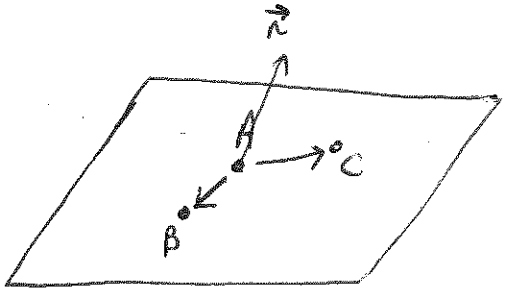


Plane

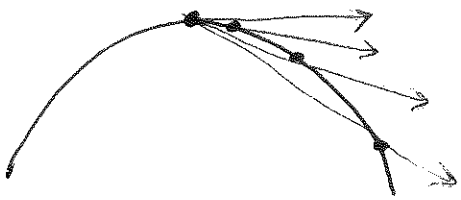


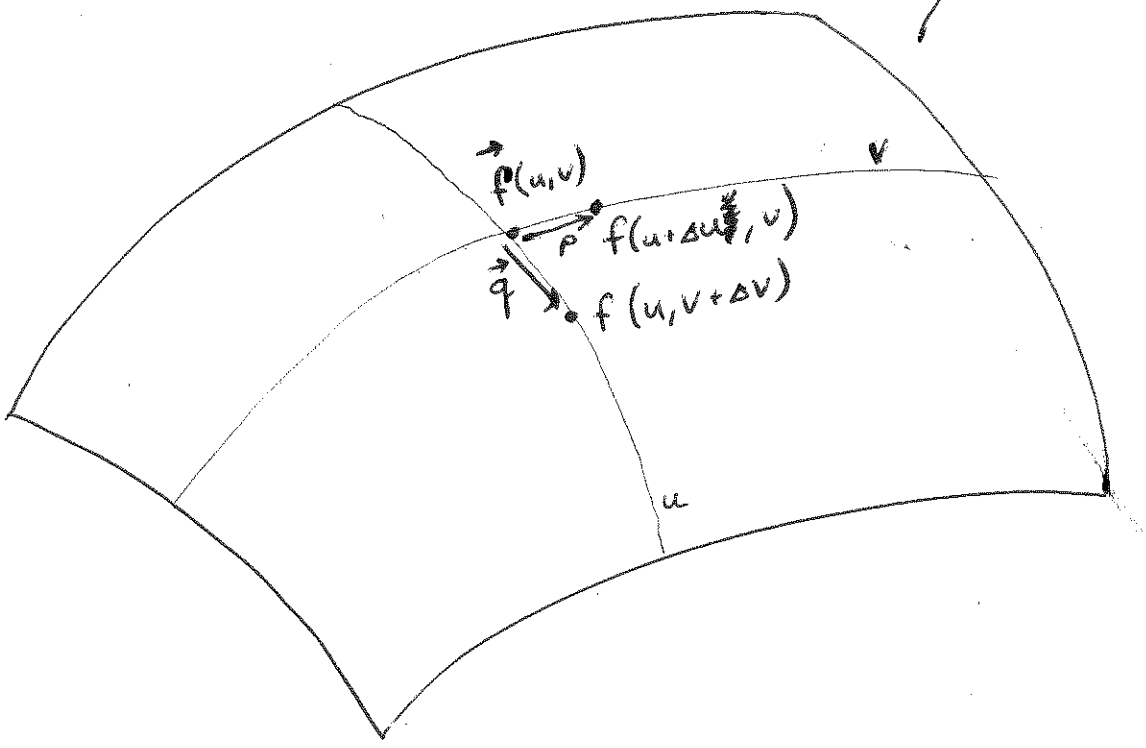
$$\vec{v} = (B-A) \times (C-A)$$

$$\vec{n} = \frac{\vec{v}}{\|\vec{v}\|}$$

Parametric

$\vec{f}(u,v)$ e.g. $f(u,v) = \begin{pmatrix} r\cos u \sin v \\ r \sin u \sin v \\ r \cos v \end{pmatrix}$





$$\vec{p} = f(u+\Delta u, v) - f(u,v)$$
$$= \left(\frac{f(u+\Delta u, v) - f(u,v)}{\Delta u}\right)\Delta u$$
$$\xrightarrow[(\Delta u \to 0)]{} \frac{\partial f}{\partial u}\,\Delta u$$
(tangent vector)

$$\vec{q} = f(u, v+\Delta v) - f(u,v)$$
$$= \left(\frac{f(u, v+\Delta v) - f(u,v)}{\Delta v}\right)\Delta v$$
$$\xrightarrow[(\Delta v \to 0)]{} \frac{\partial f}{\partial v}\,\Delta v$$
(tangent vector)

$$\vec{n} = \frac{\frac{\partial f}{\partial u} \times \frac{\partial f}{\partial v}}{\left\| \frac{\partial f}{\partial u} \times \frac{\partial f}{\partial v} \right\|}$$

Implicit

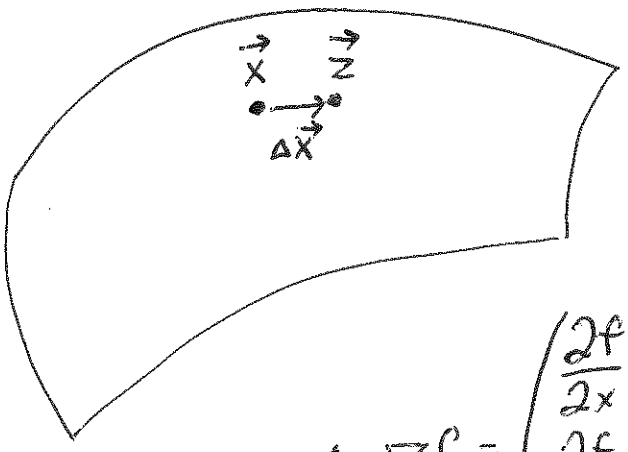
$f(\vec{x}) = 0$

plane: $(\vec{x} - \vec{p}) \cdot n = 0$

sphere: $\|\vec{x} - \vec{c}\|^2 - r^2 = 0$

$f(\vec{x}) = 0$
$f(\vec{z}) = 0$

$\vec{z} = \vec{x} + \Delta\vec{x}$



$$f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = 0$$
$$\approx \underbrace{\nabla f \cdot \Delta\vec{x}}_{=0}$$

(in 1D) $\frac{f(x+\Delta x) - f(x)}{\Delta x} \approx f'(x)\frac{\Delta x}{\Delta x}$

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$$

$\vec{n} \propto \nabla f$

$\vec{n} = \frac{\nabla f}{\|\nabla f\|}$