

$$\times \quad \frac{R^{n+1} - R^n}{\Delta t} = (\omega^*)^n R^n$$

identity

$$R^{n+1} = R^n + \Delta t (\omega^*)^n R^n \leftarrow = (\delta + \Delta t (\omega^*)^n) R^n$$

$$= R^n + O(\Delta t^2)$$

replace with
rotation

replace with $R^{n+1} = g(\Delta t \omega^n) R^n$

$$\rightarrow g(u) g(u)^T = \delta$$

$$g(\Delta t u) = \delta + \Delta t u^* + O(\Delta t^2)$$

$$g(u) = \exp(u^*) = \delta + u^* + \frac{1}{2} (u^*)^2 + \frac{1}{6} (u^*)^3 + \frac{1}{24} (u^*)^4 + \dots$$

$$g(u) = \delta + (\sin \theta) z^* + (1 - \cos \theta) (z^*)^2$$

$$\theta = \|u\| \quad z = \frac{u}{\|u\|}$$

$$\sin \theta \approx \theta - \frac{1}{6} \theta^3 + \frac{1}{120} \theta^5 - \dots$$

$$e^x = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \dots$$

$$g(\Delta t \omega) = \delta + \sin \|\Delta t \omega\| \left(\frac{\Delta t \omega}{\|\Delta t \omega\|} \right)^* + (1 - \cos \|\Delta t \omega\|) \left[\left(\frac{\Delta t \omega}{\|\Delta t \omega\|} \right)^* \right]^2$$

$$\approx \delta + \Delta t \omega^* + O(\Delta t^3) + O(\Delta t^5)$$

$$\cos \theta \approx 1 - \frac{1}{2} \theta^2 + \frac{1}{24} \theta^4 - \dots$$