Linear Operator

$$y = f(x) \qquad x \xrightarrow{f} y$$

$$f(ax + by) = af(x) + bf(y)$$

vectors

if x is a vector

$$y_i = \sum_k A_{ik} x_k \quad\Rightarrow\quad \vec{y} = A\vec{x} \qquad\qquad x \xrightarrow{A} y \xrightarrow{B} z$$

↑
Matrix

best I can do...

$$z_m = \sum_{ki} B_{mi} y_i = \sum_{ik} B_{mi} A_{ik} x_k = \sum_k \left(\sum_i B_{mi} A_{ik} \right) x_k$$

$$\underbrace{\phantom{\sum_i B_{mi} A_{ik}}}_{C_{mk}}$$

$$B(Ax) = Cx = (BA)x$$
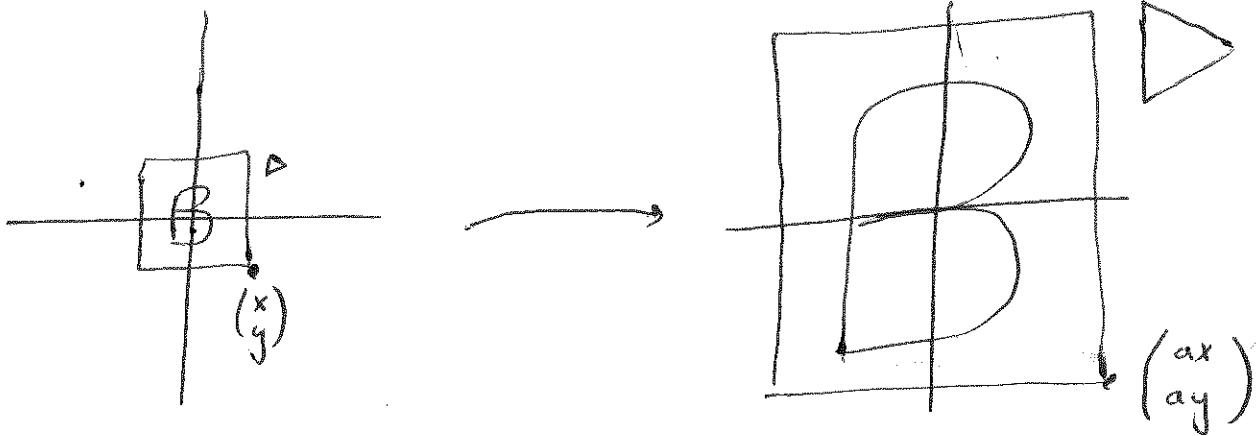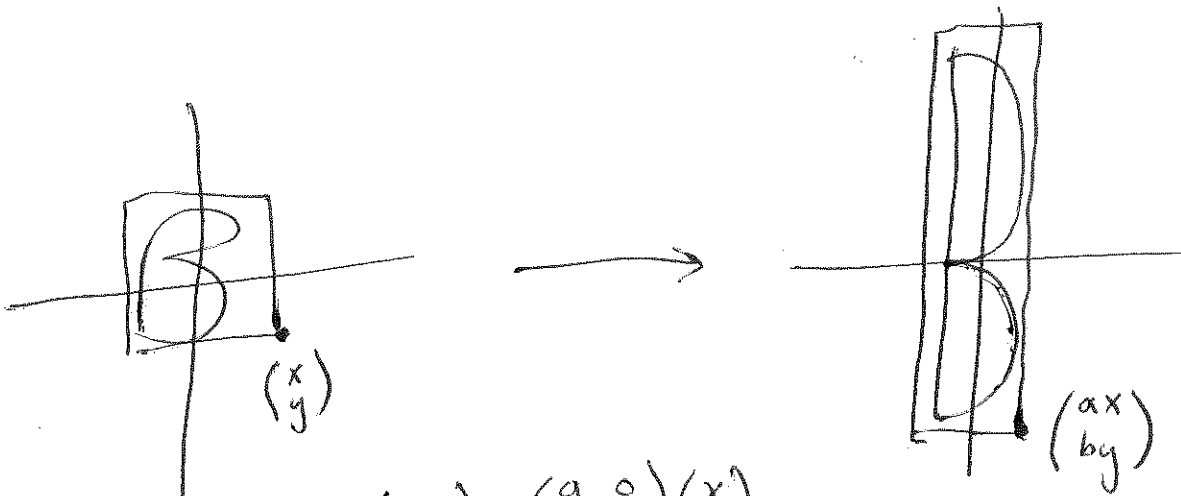
transforms

2D

translation
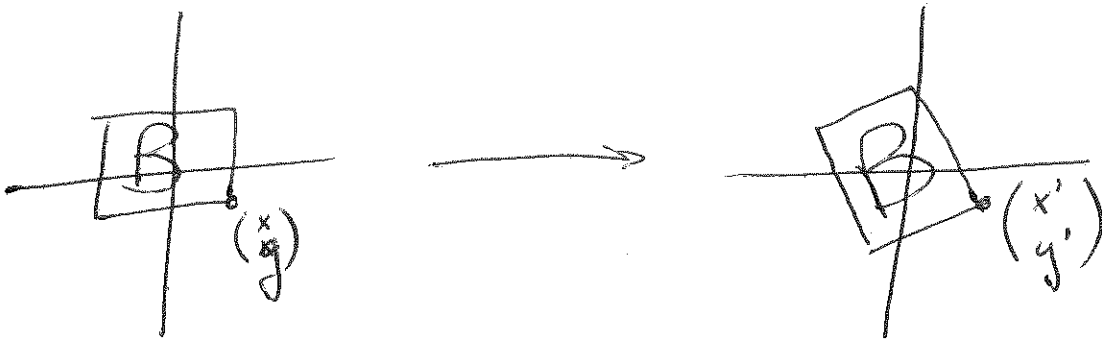
rotation

uniform scale

non-uniform scale

shear

$$\begin{pmatrix} ax \\ ay \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} ax \\ by \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\binom{x'}{y'} = \begin{pmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{pmatrix} \binom{x}{y}$$

$$\left\| \binom{x}{y} \right\|^2 = \left\| \binom{x'}{y'} \right\|^2$$

$$(x')^2 + (y')^2 = (x\cos\theta - y\sin\theta)^2 + (x\sin\theta + y\cos\theta)^2$$

$$= x^2(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) + y^2(\underbrace{\sin^2\theta + \cos^2\theta}_{1})$$

$$+ 2xy(-\cos\theta\sin\theta + \sin\theta\cos\theta)$$

$$= x^2 + y^2$$

$$\binom{x}{y} \cdot \binom{x'}{y'} = \|\vec{x}\| \, \|\vec{x'}\| \cos\theta = \|\vec{x}\|^2 \cos\theta$$

$$\underset{\vec{x}}{\uparrow} \qquad \underset{\vec{x'}}{\uparrow}$$

$$\binom{x}{y} \cdot \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{pmatrix} = x^2\cos\theta - xy\sin\theta + yx\sin\theta + y^2\cos\theta$$

$$= (x^2 + y^2)\cos\theta$$
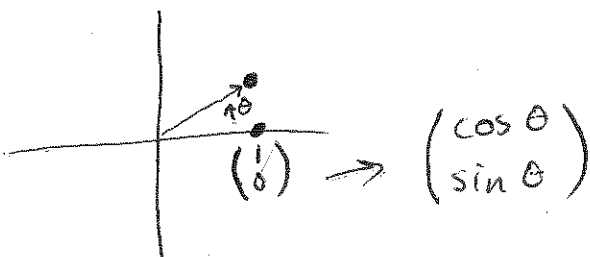
$$\binom{1}{0} \rightarrow \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

scale: $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \rightarrow \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$

2D

3D

$$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Homogeneous
Coordinates

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \longrightarrow \begin{pmatrix} x/w \\ y/w \\ z/w \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} ax \\ ay \\ az \\ aw \end{pmatrix}$$

Do not commute!