Reconstruction of implicit surfaces from fluid particles using convolutional neural networks
Technical document

Chen Zhao, Tamar Shinar and Craig Schroeder

1 Polynomial regularization

We smooth the results of our network by computing a quadratic regression and adding the residual to the loss function. To do this, we accumulate a 3^3 neighborhood of ϕ samples computed by the network into a vector Φ of size 27 in the order shown in Figure 1. Let (x_i, y_i, z_i) with $x_i, y_i, z_i \in \{-1, 0, 1\}$ be the locations of the locations of the 27 entries in Φ. Letting $\mathbf{A}$ be the Vandermonde matrix

$$\mathbf{A} = \begin{pmatrix} x_1^2 & x_1y_1 & x_1z_1 & y_1^2 & y_1z_1 & z_1^2 & x_1 & y_1 & z_1 & 1 \\ x_2^2 & x_2y_2 & x_2z_2 & y_2^2 & y_2z_2 & z_2^2 & x_2 & y_2 & z_2 & 1 \\ \vdots \\ x_{27}^2 & x_{27}y_{27} & x_{27}z_{27} & y_{27}^2 & y_{27}z_{27} & z_{27}^2 & x_{27} & y_{27} & z_{27} & 1 \end{pmatrix}$$

we wish to choose coefficients C for our polynomial so that $\|\mathbf{A}C - \Phi\|_2^2$ is minimized. These coefficients are given by $C = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\Phi$. Then, the error is $L_p(\Phi) = \|\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\Phi - \Phi\|_2^2$. Defining the projection operator $\mathbf{K} = \mathbf{I} - \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ the polynomial fit error is simply

$$L_p(\Phi) = \|\mathbf{K}\Phi\|_2^2 = \Phi^T\mathbf{K}\Phi. \tag{1}$$

The matrix $\mathbf{K}$ is constant and hard-coded. The specific sample positions and corresponding $\mathbf{A}$ are:

i	x_i	y_i	z_i
1	-1	-1	-1
2	0	-1	-1
3	1	-1	-1
4	-1	0	-1
5	0	0	-1
6	1	0	-1
7	-1	1	-1
8	0	1	-1
9	1	1	-1
10	-1	-1	0
11	0	-1	0
12	1	-1	0
13	-1	0	0
14	0	0	0
15	1	0	0
16	-1	1	0
17	0	1	0
18	1	1	0
19	-1	-1	1
20	0	-1	1
21	1	-1	1
22	-1	0	1
23	0	0	1
24	1	0	1
25	-1	1	1
26	0	1	1
27	1	1	1

$$\mathbf{A} = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \\
1 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & -1 & 1 \\
1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
0 & 0 & 0 & 1 & -1 & 1 & 0 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \\
1 & -1 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & -1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
0 & 0 & 0 & 1 & -1 & 1 & 0 & -1 & 1 & 1 \\
1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 \\
1 & 0 & -1 & 0 & 0 & 1 & -1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

With these, the matrix **K** is.

$$
\mathbf{K} = \mathbf{I} - \mathbf{A}(\mathbf{A}^T A)^{-1}\mathbf{A}^T =
$$

$$
\frac{1}{108}\begin{pmatrix}
53 & -25 & -7 & -25 & -4 & 5 & -7 & 5 & 5 & -25 & -4 & 5 & -4 & 8 & 8 & 5 & 8 & -1 & -7 & 5 & 5 & 5 & 8 & -1 & 5 & -1 & -19 \\
-25 & 71 & -25 & -4 & -16 & -4 & 5 & -7 & 5 & -4 & -16 & -4 & 8 & -4 & 8 & 8 & -4 & 8 & 5 & -7 & 5 & 8 & -4 & 8 & -1 & -13 & -1 \\
-7 & -25 & 53 & 5 & -4 & -25 & 5 & 5 & -7 & 5 & -4 & -25 & 8 & 8 & -4 & -1 & 8 & 5 & 5 & 5 & -7 & -1 & 8 & 5 & -19 & -1 & 5 \\
-25 & -4 & 5 & 71 & -16 & -7 & -25 & -4 & 5 & -4 & 8 & 8 & -16 & -4 & -4 & -4 & 8 & 8 & 5 & 8 & -1 & -7 & -4 & -13 & 5 & 8 & -1 \\
-4 & -16 & -4 & -16 & 80 & -16 & -4 & -16 & -4 & 8 & -4 & 8 & -4 & -16 & -4 & 8 & -4 & 8 & 8 & -4 & 8 & -4 & -16 & -4 & 8 & -4 & 8 \\
5 & -4 & -25 & -7 & -16 & 71 & 5 & -4 & -25 & 8 & 8 & -4 & -4 & -4 & -16 & 8 & 8 & -4 & -1 & 8 & 5 & -13 & -4 & -7 & -1 & 8 & 5 \\
-7 & 5 & 5 & -25 & -4 & 5 & 53 & -25 & -7 & 5 & 8 & -1 & -4 & 8 & 8 & -25 & -4 & 5 & 5 & -1 & -19 & 5 & 8 & -1 & -7 & 5 & 5 \\
5 & -7 & 5 & -4 & -16 & -4 & -25 & 71 & -25 & 8 & -4 & 8 & 8 & -4 & 8 & -4 & -16 & -4 & -1 & -13 & -1 & 8 & -4 & 8 & 5 & -7 & 5 \\
5 & 5 & -7 & 5 & -4 & -25 & -7 & -25 & 53 & -1 & 8 & 5 & 8 & 8 & -4 & 5 & -4 & -25 & -19 & -1 & 5 & -1 & 8 & 5 & 5 & 5 & -7 \\
-25 & -4 & 5 & -4 & 8 & 8 & 5 & 8 & -1 & 71 & -16 & -7 & -16 & -4 & -4 & -7 & -4 & -13 & -25 & -4 & 5 & -4 & 8 & 8 & 5 & 8 & -1 \\
-4 & -16 & -4 & 8 & -4 & 8 & 8 & -4 & 8 & -16 & 80 & -16 & -4 & -16 & -4 & -4 & -16 & -4 & -4 & -16 & -4 & 8 & -4 & 8 & 8 & -4 & 8 \\
5 & -4 & -25 & 8 & 8 & -4 & -1 & 8 & 5 & -7 & -16 & 71 & -4 & -4 & -16 & -13 & -4 & -7 & 5 & -4 & -25 & 8 & 8 & -4 & -1 & 8 & 5 \\
-4 & 8 & 8 & -16 & -4 & -4 & -4 & 8 & 8 & -16 & -4 & -4 & 80 & -16 & -16 & -16 & -4 & -4 & -4 & 8 & 8 & -16 & -4 & -4 & -4 & 8 & 8 \\
8 & -4 & 8 & -4 & -16 & -4 & 8 & -4 & 8 & -4 & -16 & -4 & -16 & 80 & -16 & -4 & -16 & -4 & 8 & -4 & 8 & -4 & -16 & -4 & 8 & -4 & 8 \\
8 & 8 & -4 & -4 & -4 & -16 & 8 & 8 & -4 & -4 & -4 & -16 & -16 & -16 & 80 & -4 & -4 & -16 & 8 & 8 & -4 & -4 & -4 & -16 & 8 & 8 & -4 \\
5 & 8 & -1 & -4 & 8 & 8 & -25 & -4 & 5 & -7 & -4 & -13 & -16 & -4 & -4 & 71 & -16 & -7 & 5 & 8 & -1 & -4 & 8 & 8 & -25 & -4 & 5 \\
8 & -4 & 8 & 8 & -4 & 8 & -4 & -16 & -4 & -4 & -16 & -4 & -4 & -16 & -4 & -16 & 80 & -16 & 8 & -4 & 8 & 8 & -4 & 8 & -4 & -16 & -4 \\
-1 & 8 & 5 & 8 & 8 & -4 & 5 & -4 & -25 & -13 & -4 & -7 & -4 & -4 & -16 & -7 & -16 & 71 & -1 & 8 & 5 & 8 & 8 & -4 & 5 & -4 & -25 \\
-7 & 5 & 5 & 5 & 8 & -1 & 5 & -1 & -19 & -25 & -4 & 5 & -4 & 8 & 8 & 5 & 8 & -1 & 53 & -25 & -7 & -25 & -4 & 5 & -7 & 5 & 5 \\
5 & -7 & 5 & 8 & -4 & 8 & -1 & -13 & -1 & -4 & -16 & -4 & 8 & -4 & 8 & 8 & -4 & 8 & -25 & 71 & -25 & -4 & -16 & -4 & 5 & -7 & 5 \\
5 & 5 & -7 & -1 & 8 & 5 & -19 & -1 & 5 & 5 & -4 & -25 & 8 & 8 & -4 & -1 & 8 & 5 & -7 & -25 & 53 & 5 & -4 & -25 & 5 & 5 & -7 \\
5 & 8 & -1 & -7 & -4 & -13 & 5 & 8 & -1 & -4 & 8 & 8 & -16 & -4 & -4 & -4 & 8 & 8 & -25 & -4 & 5 & 71 & -16 & -7 & -25 & -4 & 5 \\
8 & -4 & 8 & -4 & -16 & -4 & 8 & -4 & 8 & 8 & -4 & 8 & -4 & -16 & -4 & 8 & -4 & 8 & -4 & -16 & -4 & -16 & 80 & -16 & -4 & -16 & -4 \\
-1 & 8 & 5 & -13 & -4 & -7 & -1 & 8 & 5 & 8 & 8 & -4 & -4 & -4 & -16 & 8 & 8 & -4 & 5 & -4 & -25 & -7 & -16 & 71 & 5 & -4 & -25 \\
5 & -1 & -19 & 5 & 8 & -1 & -7 & 5 & 5 & 5 & 8 & -1 & -4 & 8 & 8 & -25 & -4 & 5 & -7 & 5 & 5 & -25 & -4 & 5 & 53 & -25 & -7 \\
-1 & -13 & -1 & 8 & -4 & 8 & 5 & -7 & 5 & 8 & -4 & 8 & 8 & -4 & 8 & -4 & -16 & -4 & 5 & -7 & 5 & -4 & -16 & -4 & -25 & 71 & -25 \\
-19 & -1 & 5 & -1 & 8 & 5 & 5 & 5 & -7 & -1 & 8 & 5 & 8 & 8 & -4 & 5 & -4 & -25 & 5 & 5 & -7 & 5 & -4 & -25 & -7 & -25 & 53
\end{pmatrix}
$$

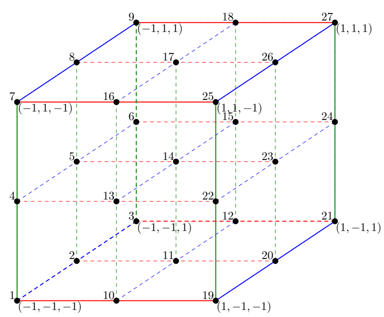

Figure 1: Cluster of ϕ values on subgrid

References