CS/EE 217 GPU Architecture and Parallel Programming

Lecture 11

Parallel Computation Patterns – Parallel Prefix Sum (Scan)

Objective

- To master parallel Prefix Sum (Scan) algorithms
  - frequently used for parallel work assignment and resource allocation
  - A key primitive to in many parallel algorithms to convert serial computation into parallel computation
  - Based on reduction tree and reverse reduction tree

- Reading –Efficient Parallel Scan Algorithms for GPUs
  - https://research.nvidia.com/sites/default/files/publications/nvr-2008-003.pdf

(Inclusive) Prefix-Sum (Scan) Definition

Definition: *The* all-prefix-sums *operation takes a binary associative operator* $\oplus$*, and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the array

$$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, then the all-prefix-sums operation on the array [3 1 7 0 4 1 6 3], would return [3 4 11 11 15 16 22 25].

A Inclusive Scan Application Example

- Assume that we have a 100-inch sausage to feed 10
- We know how much each person wants in inches
  - [3 5 2 7 28 4 3 0 8 1]
- How do we cut the sausage quickly?
- How much will be left

- Method 1: cut the sections sequentially: 3 inches first, 5 inches second, 2 inches third, etc.
- Method 2: calculate Prefix scan
  - [3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 inches left)

Typical Applications of Scan

- Scan is a simple and useful parallel building block
  - Convert recurrences from sequential :

```
for(j=1;j<n;j++)
    out[j] = out[j-1] + f(j);
```

  - into parallel:

```
forall(j) { temp[j] = f(j) };
scan(out, temp);
```

- Useful for many parallel algorithms:
  - radix sort
  - quicksort
  - String comparison
  - Lexical analysis
  - Stream compaction
  - Polynomial evaluation
  - Solving recurrences
  - Tree operations
  - Histograms
  - Etc.

Other Applications

- Assigning camp slots
- Assigning farmer market space
- Allocating memory to parallel threads
- Allocating memory buffer for communication channels
- …

An Inclusive Sequential Scan

Given a sequence $[x_0, x_1, x_2, \ldots]$

Calculate output $[y_0, y_1, y_2, \ldots]$

Such that $y_0 = x_0$

$$y_1 = x_0 + x_1$$

$$y_2 = x_0 + x_1 + x_2$$

…

Using a recursive definition

$$y_i = y_{i-1} + x_i$$

A Work Efficient C Implementation

```
y[0] = x[0];
for (i = 1; i < Max_i; i++) y[i] = y [i-1] + x[i];
```

Computationally efficient:

N additions needed for N elements - O(N)!

How do we do this in parallel?

- What is the relationship between Parallel Scan and Reduction?
  - Multiple reduction operations!
  - What if we implement it that way?

- How many operations?
  - Each reduction tree is O(n) operations
  - Reduction trees of size n, n-1, n-2, n-3, … 1
  - Very work inefficient! Important concept

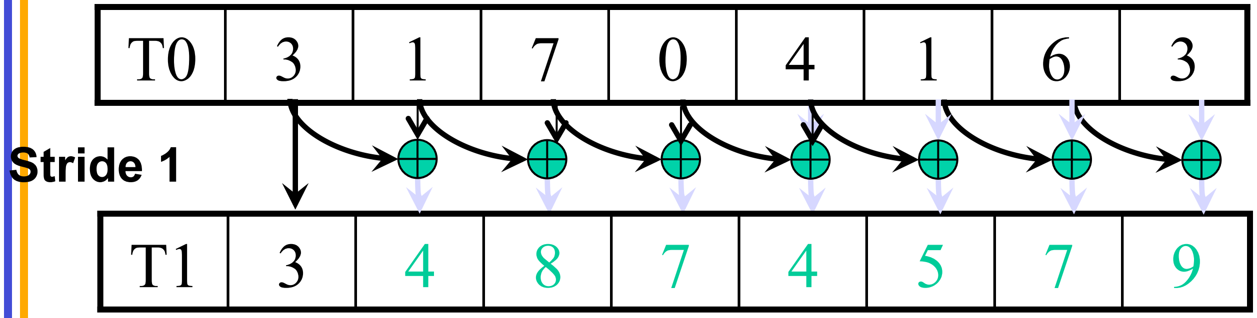
A Slightly Better Parallel Inclusive Scan Algorithm

T0	3	1	7	0	4	1	6	3

1. Read input from device memory to shared memory

Each thread reads one value from the input array in device memory into shared memory array T0. Thread 0 writes 0 into shared memory array.

A Slightly Better Parallel Scan Algorithm



T0	3	1	7	0	4	1	6	3
T1	3	4	8	7	4	5	7	9

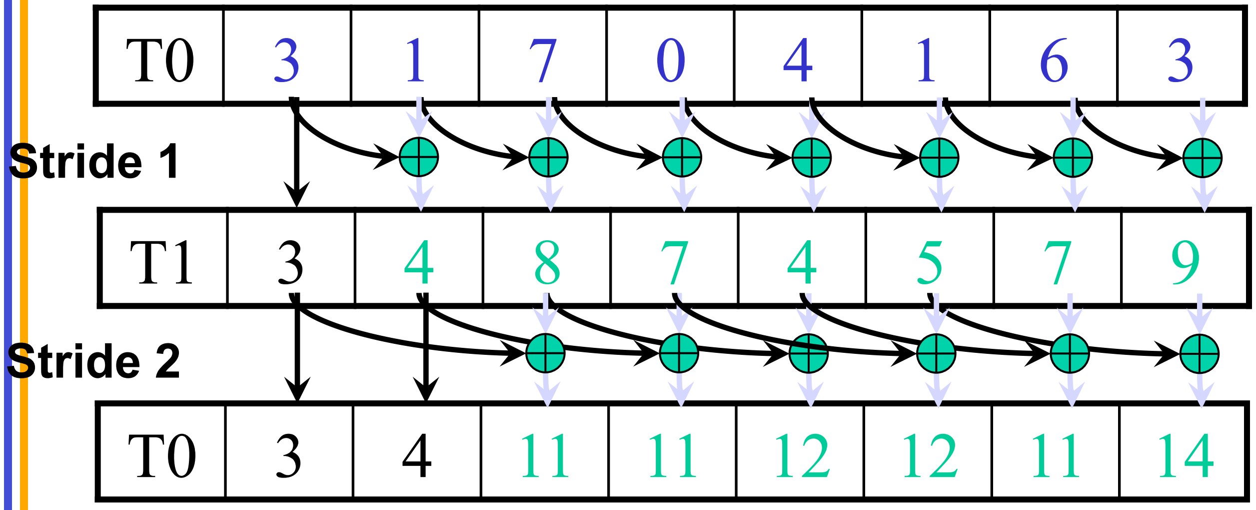
Stride 1

1. (previous slide)
2. Iterate log(n) times: Threads *stride* to *n:* Add pairs of elements *stride* elements apart. Double *stride* at each iteration. (note must double buffer shared mem arrays)

Iteration #1
Stride = 1

- Active threads: *stride* to *n*-1 (*n-stride* threads)
- Thread *j* adds elements *j* and *j-stride* from T0 and writes result into shared memory buffer T1 (ping-pong)

A Slightly Better Parallel Scan Algorithm

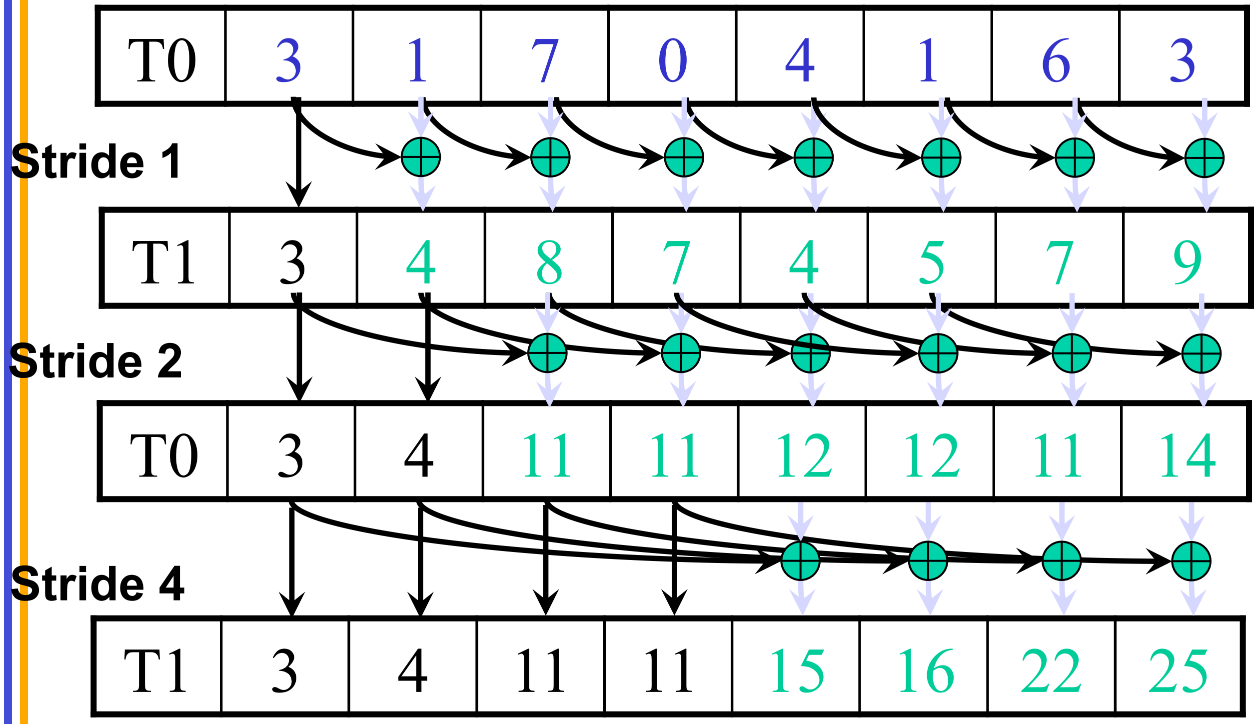


T0	3	1	7	0	4	1	6	3
T1	3	4	8	7	4	5	7	9
T0	3	4	11	11	12	12	11	14

1. Read input from device memory to shared memory.
2. Iterate log(n) times: Threads *stride* to *n:* Add pairs of elements *stride* elements apart. Double *stride* at each iteration. (note must double buffer shared mem arrays)

Iteration #2
Stride = 2

A Slightly Better Parallel Scan Algorithm



Iteration #3
Stride = 4

1. Read input from device memory to shared memory. Set first element to zero and shift others right by one.
2. Iterate log(n) times: Threads *stride* to *n:* Add pairs of elements *stride* elements apart. Double *stride* at each iteration. (note must double buffer shared memory arrays)
3. Write output from shared memory to device memory

Handling Dependencies

- During every iteration, each therad can overwrite the input of another thread
  - Need barrier synchronization to ensure all inputs have been properly generated
  - All threads secure input operand that can be overwritten by another thread
  - Barrier synchronization to ensure that threads have secured their inputs
  - All threads perform addition to write output

Work inefficient scan kernel

```
__shared__ float   XY[SECTION_SIZE];
int i = blockIdx.x * blockDim.x + threadIdx.x;

//load into shared memory
if (i < InputSize) { XY[threadIdx.x] = X[i];}

//perform iterative scan on XY
for (unsigned int stride = 1; stride <= threadIdx.x; stride *=2) {
            __synchthreads();
            float in1 = XY[threadIdx.x – stride];
            __synchthreads();
            XY[threadIdx.x]+=in1;
}
```

Work Efficiency Considerations

- The first-attempt Scan executes log(n) parallel iterations
  - The steps do (n-1), (n-2), (n-4),..(n- n/2) adds each
  - Total adds: n * log(n) - (n-1) → O(n*log(n)) work
- This scan algorithm is not very work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) hurts: 20x for 10^6 elements!
- A parallel algorithm can be slow when execution resources are saturated due to low work efficiency

Improving Efficiency

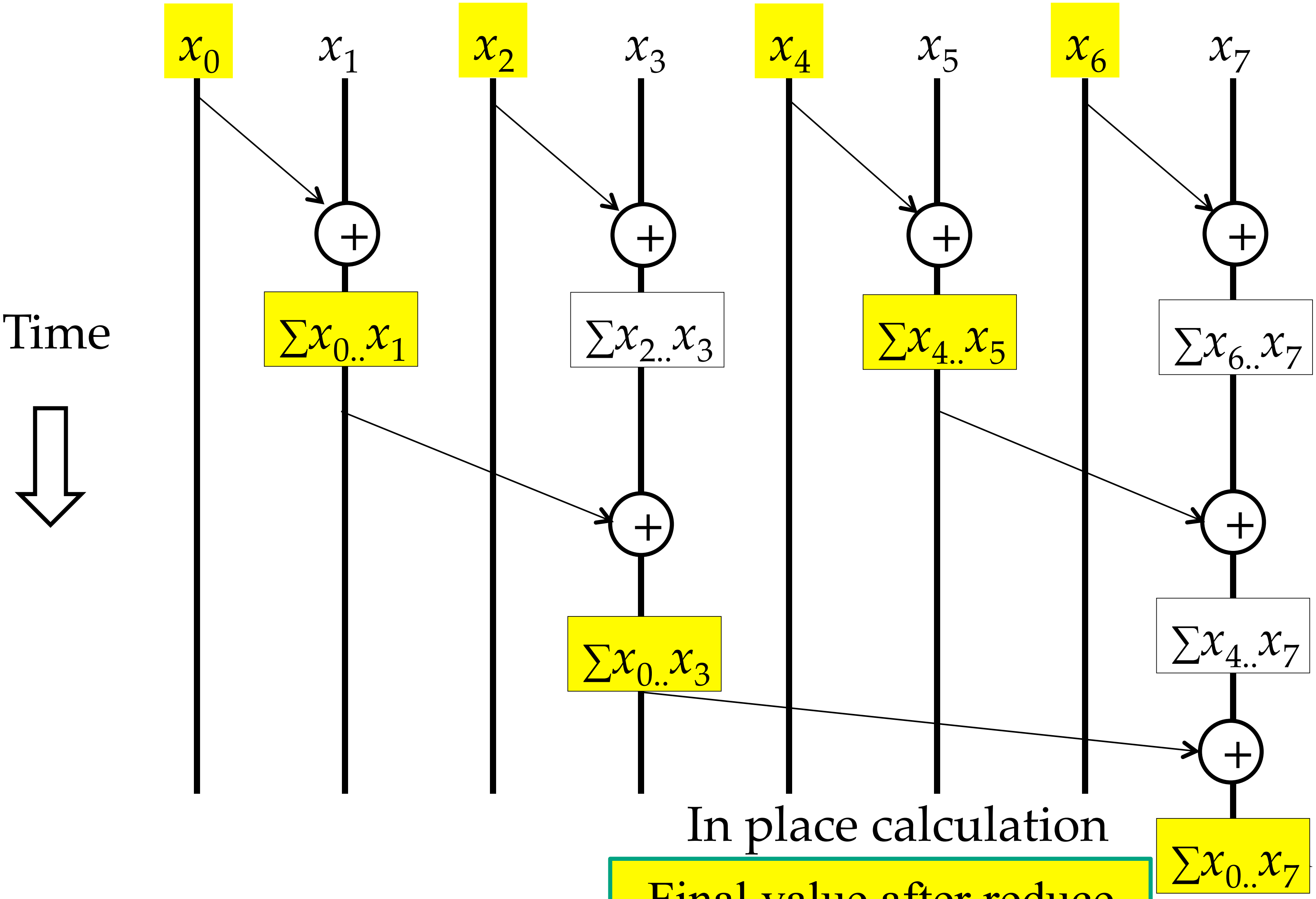
- A common parallel algorithm pattern:

  Balanced Trees

  - Build a balanced binary tree on the input data and sweep it to and from the root
  - Tree is not an actual data structure, but a concept to determine what each thread does at each step

- For scan:
  - Traverse down from leaves to root building partial sums at internal nodes in the tree
    - Root holds sum of all leaves
  - Traverse back up the tree building the scan from the partial sums

Parallel Scan - Reduction Step



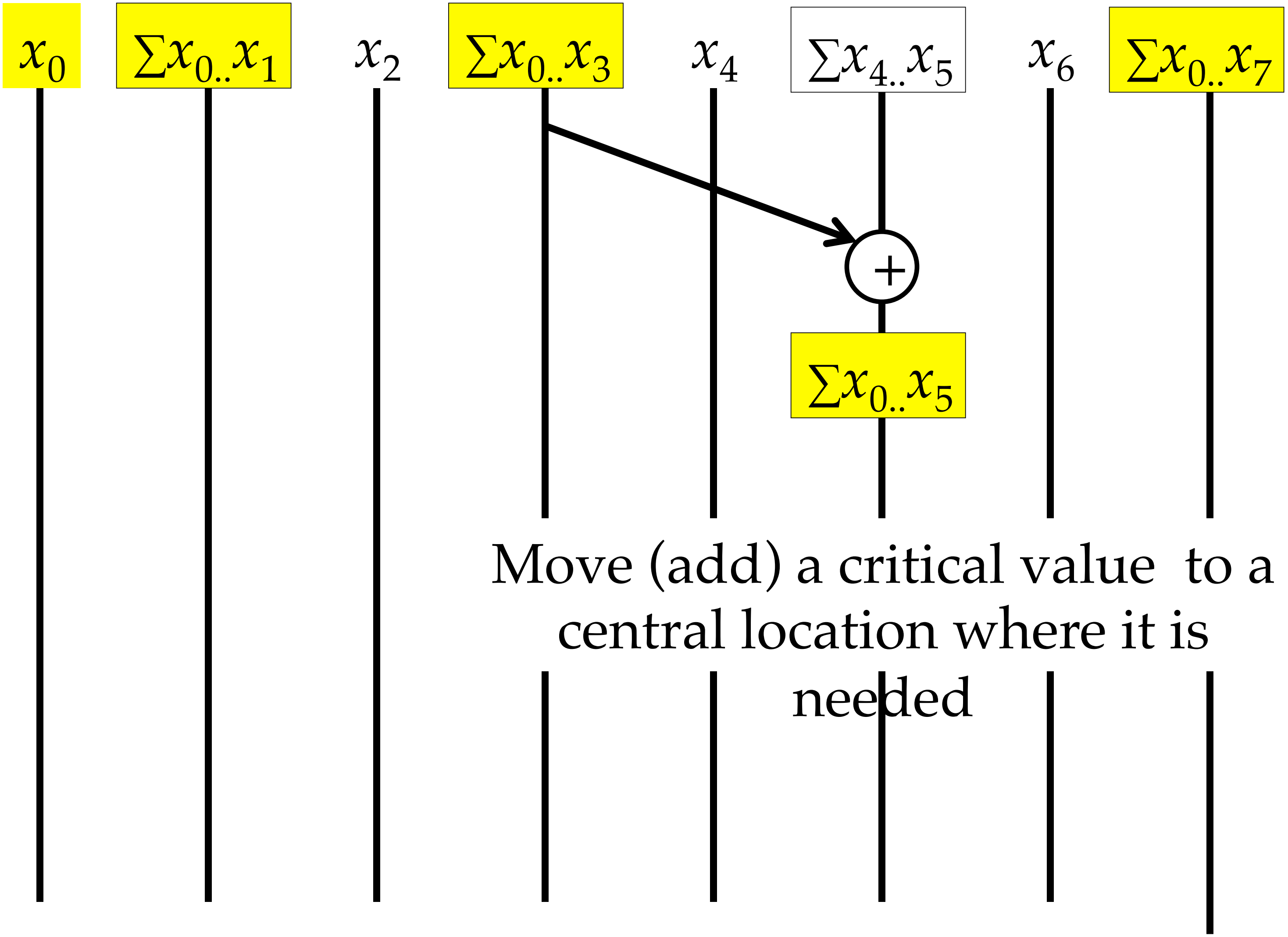
In place calculation

Final value after reduce

Reduction Step Kernel Code

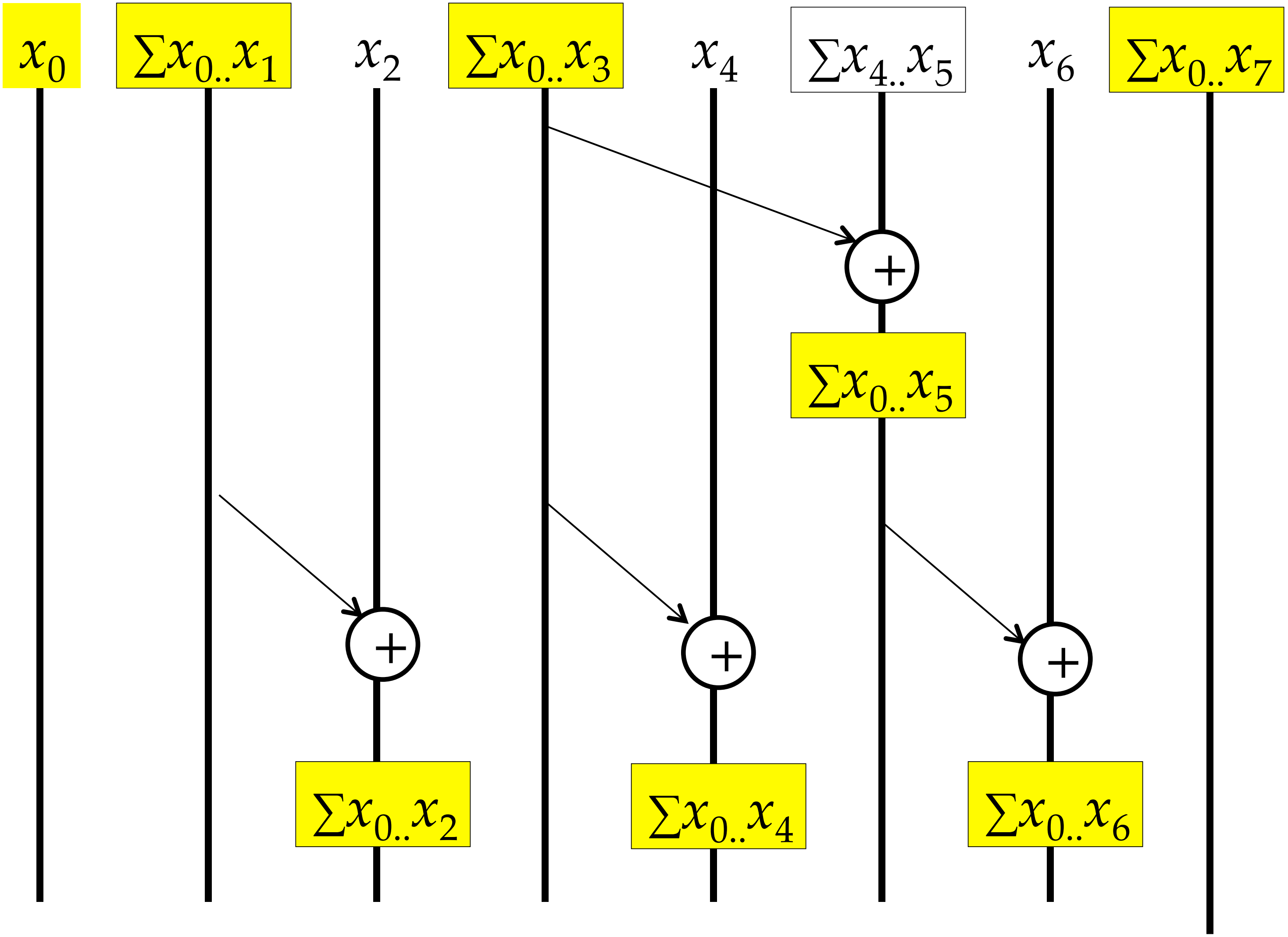
```
 // scan_array[BLOCK_SIZE*2] is in shared memory

for(int stride=1; stride<= BLOCK_SIZE; stride *=2)
   {
       int index = (threadIdx.x+1)*stride*2 - 1;
       if(index < 2*BLOCK_SIZE)
           scan_array[index] += scan_array[index-stride];
       stride = stride*2;

       __syncthreads();
   }
```

Inclusive Post Scan Step

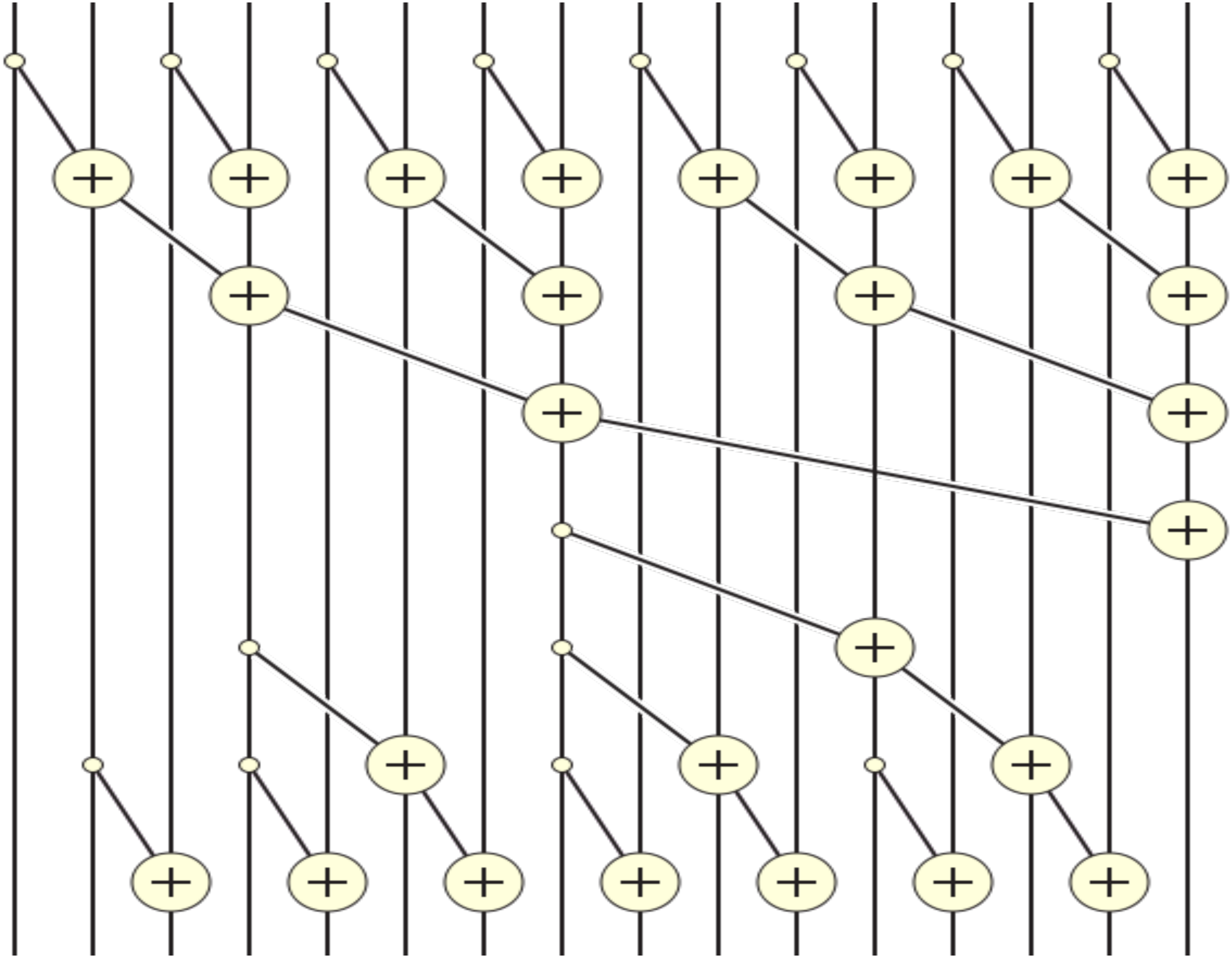


Move (add) a critical value to a central location where it is needed

Inclusive Post Scan Step



Putting it Together



ANY MORE QUESTIONS?